Prediction of Cutoff Frequency of Irregular Pentagonal Waveguide

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Abstract

This article uses the conformal mapping technique (CMT) to study the irregular pentagonal waveguide (IPWG). This technique is applied to map the IPWG into a simple rectangular one. The lowest cutoff frequency (f_c) of this waveguide is calculated from its wave equation in the transformed domain using the Galerkin method. The relation between k and β of the pentagonal waveguide is also studied with the help of this technique. Numerical 3D electromagnetic simulator (HFSS) is used to validate the results.

1. Introduction

Extremely high microwave power is required to transfer through the waveguide for the radar and the communication system. The power handling capability is one of the essential factors for waveguide. The dielectric material in the waveguide improves power handling capability. But this high power transmission can be hampered by the physical collapse of the dielectric material. The breakpoint of dielectric material can be increased by using high-pressure gas as a dielectric material. The power handling capability depends on the waveguide's cross-section and size. The rectangular and elliptical waveguides have an excellent power handling capability. The best waveguide for handling high power is a pentagonal one [1]. It can be viewed as a combined property of elliptical and rectangular shape waveguides. But regular shape pentagonal waveguide is insufficient to achieve optimum power handling capability. Therefore, theoretical development is required for optimal engineering applications. This waveguide can also be used as a high-power device like a couple, isolator, circulator, resonator, and attenuator. The eigenmode-expansion method [2] with the point matching technique has been used to study the pentagonal waveguide. This method is limited to calculating accurate cutoff frequency. In [3], Ritz method has also been used to investigate the pentagonal waveguide for the TM mode only. Commercial 3D EM simulation software has also been used to examine the complicated waveguides [4, 5], but they need a high execution time.

CMT is a great tool for analyzing complicated waveguides. This technique helps to map the complex geometry to a simple shape where the boundary condition can be applied smoothly. Different ridge waveguides like elliptical, semielliptical, semicircular, and trapezoidal have also been studied extensively [6, 7]. Researchers can determine the different parameters of a waveguide [8-10] with the help of this method. Conformal mapping technique has been used to analyze the groove waveguides in [11]. It has also been used to determine the impedance of various microstrip lines. [12, 13].

In this manuscript, an irregular pentagonal waveguide (IPWG) is mapped to a rectangle with the help of a proper mapping function. Galarking method is utilized to determine the eigenvalues and eigenfunctions from the wave equation in the transformed domain. The cutoff frequency (f_c) of the irregular pentagonal waveguide is determined using our theoretical approach and validated with the 3D electromagnetic simulator data. The $k-\beta$ diagram of the irregular pentagonal waveguide is also plotted using CMT. Our theoretical results are validated with the help of the HFSS result.

2. Theory

Conformal mapping technique is an excellent tool for simplifying complex geometries. For the theoretical analysis, irregular pentagon is represented by one complex $w(=x+iy)$-plane. This original structure is mapped into a rectangular one using a proper mapping function, as shown in Eq. (1) [14]:

$$w = f(t) = C_o \int_o^t [cn\,(t,k)]^{\frac{2}{N}}\, dt \tag{1}$$

Typically, N is a positive integer greater than two. Where,

$$C_o = a / \int_o^K [cn\,(t,k)]^{\frac{2}{N}}\, dt$$

Here, the transformed shape represents another complex $t(=u+iv)$ -plane. $Cn(w,k)$ is the Jacobian elliptic function [15] with complex variable w and modulus k. Here, $K(k)$ represents the complete elliptic integral of modulus k. The value of k can be obtained as:

$$\frac{b}{a} = \frac{K'(k)}{K(k)} \quad (2)$$

Here; a and b represent the length of the irregular pentagonal side, as shown in Fig.1. Fig. 1 shows the mapping of all points of the irregular pentagon (w-plane) to rectangular one (t-plane).

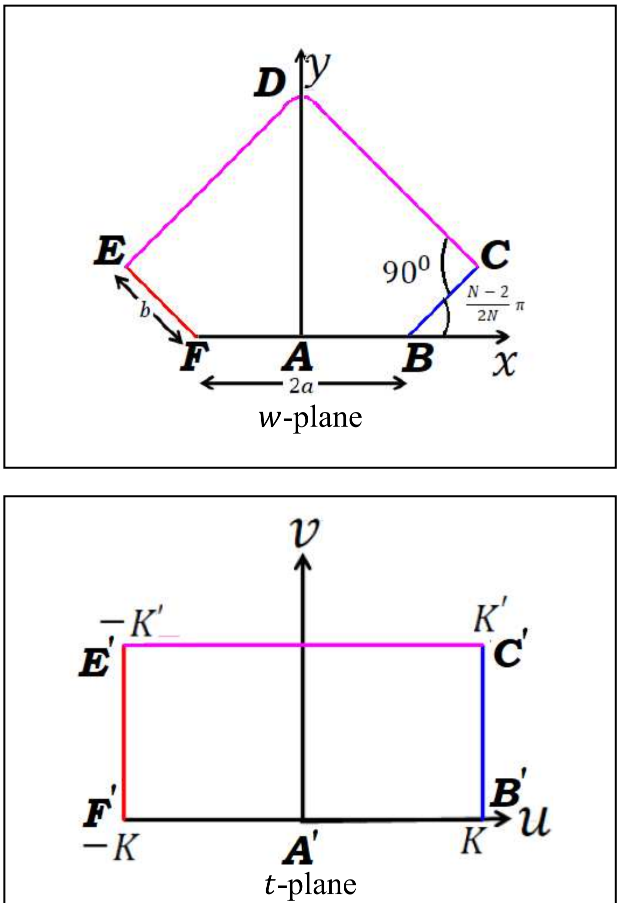

Fig. 1. Mapping from w-plane to t-plane

2.1 Wave Equation in Transform domain

After the transformation of the irregular pentagonal shape to a rectangle one, the wave equation also needs to be transformed. The wave equation in the original domain (w-plane) is given by [17]:

$$\nabla^2\psi(x,y,z) + k^2\psi(x,y,z) = 0 \quad (3)$$

As the mapping function will be applied along the (x, y) plane, there will be no change along the z-axis. Therefore, the wave-equation in w-plane can be written as;

$$\nabla_t^2\psi_1(x,y) + k_c^2\psi_1(x,y) = 0 \quad (w-\text{plane}) \quad (4)$$

where, $\frac{1}{Z(z)}\frac{\partial^2 Z(z)}{\partial z^2} = -k_z^2$

The transformed wave equation is obtained in the t-plane with the help of the Cauchy Riemann Theorem [18], which can be expressed as:

$$\nabla_t^2\psi_2(u,v) + \lambda h^2\psi_2(u,v) = 0 \quad (t-\text{plane}) \quad (5)$$

where, $h = |cn\,(w,k)|^{4/N}$ and $\lambda = (k_cC_o)^2$ and other terms carrying their conventional meaning. Therefore, k_c can be computed as

$$k_c = \sqrt{\lambda/C_o} \quad (6)$$

2.2 Eigenvalue and Eigenfunction

Galerkin method [16] is used to find the approximate solution of Eq. (5). It is assumed as:

$$\breve{\psi}_2(u,v) = a_1f_1(u,v) + a_2f_2(u,v) + \cdots + a_qf_q(u,v) \quad (7)$$

Here, a_q is the expansion coefficient and $f_q(u,v)$ is the basis function. $f_q(u,v)$ should satisfy the boundary condition for TE mode in the transformed domain. Here, the basis function is assumed as $f_q(u,v) = \cos\left[k_u\left(u-\frac{L}{2}\right)\right]\cos[k_vv]$, where, $k_u = \frac{m\pi}{2K}$; $k_v = \frac{n\pi}{K'}$. Substituting $\breve{\psi}_2(\rho,\varphi)$ into Eq. (5) and taking the inner product with respect to the weighting function w_g, we obtain:

$$[l_{gq}][a_q] = \lambda[e_{gq}][a_q] \quad (8)$$

where, $[l_{gq}] = \langle w_g, L\,(u_q)\rangle$ and $[e_{gq}] = \langle w_g, M(u_q)\rangle$ $L = -\nabla^2$ and $M = |h\,(\rho,\varphi)|^2$ where, λ is the eigenvalue. Here, weighting (w_g) and basis (u_q) both functions are identical. The q and g are integers specified as a combination of m and n. Eq. (5) has the solution if

$$det|l_{gq} - \lambda\,e_{gq}| = 0 \quad (9)$$

After solving Eq. (9) for the lowest value of $\lambda = \lambda_1$, we can easily find k_c from eq. (6). The cutoff frequency (f_c) of the irregular pentagonal shape waveguide is obtained as [17]:

$$f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} \quad (10)$$

3. Cutoff Frequency and Field patterns

Eq. (10) can be used to determine the cutoff frequency (f_c) of the irregular pentagonal waveguide. Our theoretical results and simulated data from a 3D Electromagnetic simulator are compared as shown in Table 1. A good agreement is found between our results and data obtained using 3D electromagnetic simulator HFSS.

TABLE 1 COMPARISON OF CUTOFF FREQUENCY WITH SIMULATED DATA (HFSS)

a	b	Frequency (GHz)		Error
mm	mm	HFSS[19]	Theory	(%)
75	150	0.575	0.572	0.5217
60	42	1.09	1.11	1.8349
50	75	0.987	0.997	1.0132
25	50	1.712	1.717	0.2921
20	20	2.92	2.95	1.0274
15	20	3.475	3.50	0.7194
10	10	5.840	5.90	1.0274
5	20	5.5	5.6	1.8182
36	34	1.65	1.674	1.4545

The internal $\vec{E}$ and $\vec{H}$ field patterns for the fundamental TE mode are also plotted. Fig. 2 shows the internal $\vec{E}$ field pattern of the irregular pentagonal waveguide and compared with HFSS result.

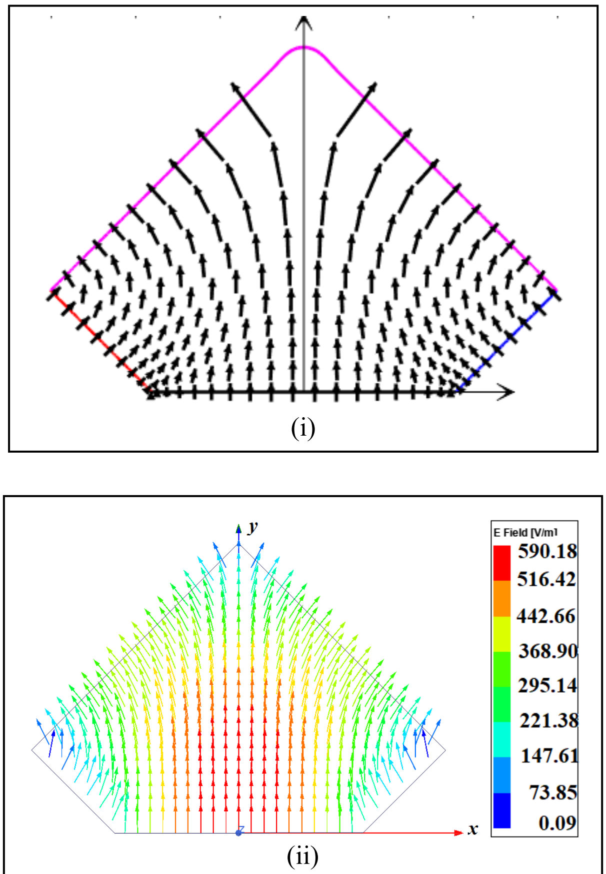

Fig. 2. Internal $\vec{E}$ -field distribution for dominant TE mode for $a = 36\ mm, b = 34\ mm$ (i) theory (ii) HFSS

Similarly, the internal $\vec{H}$ field is also plotted using our theory and validated with 3D EM simulated data. This is shown in Fig. 3. It is found that our theory for plotting internal fields ($\vec{E}$ and $\vec{H}$) agrees well with 3D electromagnetic simulator HFSS data.

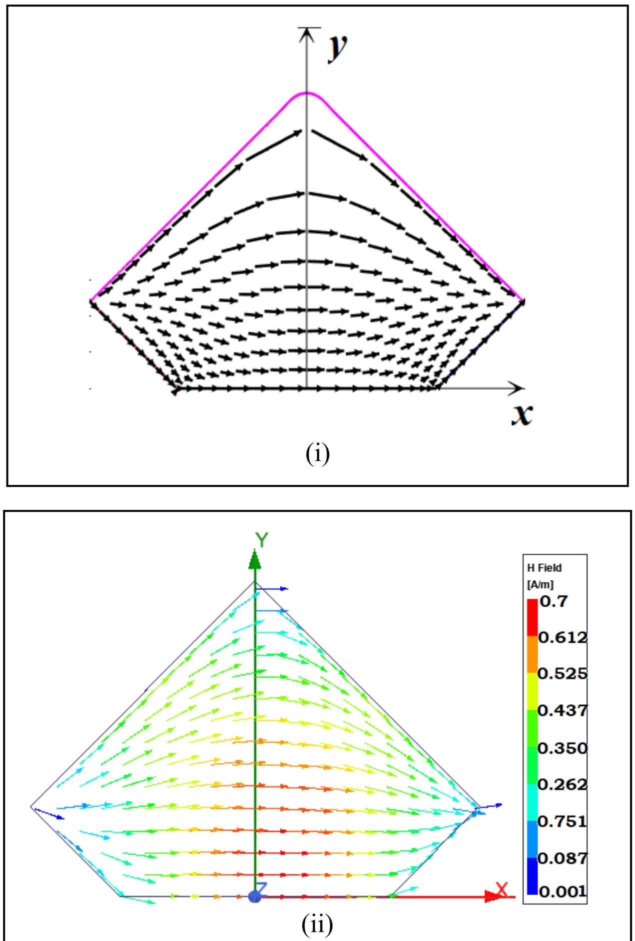

Fig. 3. Internal $\vec{H}$ -field distribution for the fundamental TE mode for $a = 36\ mm, b = 34\ mm$ (i) theory (ii) HFSS

4. Dispersion diagram

The Dispersion diagram of the irregular pentagonal waveguide is computed using the CMT. Therefore, β as a function of k is plotted using the CMT. This is shown in Fig. 4.

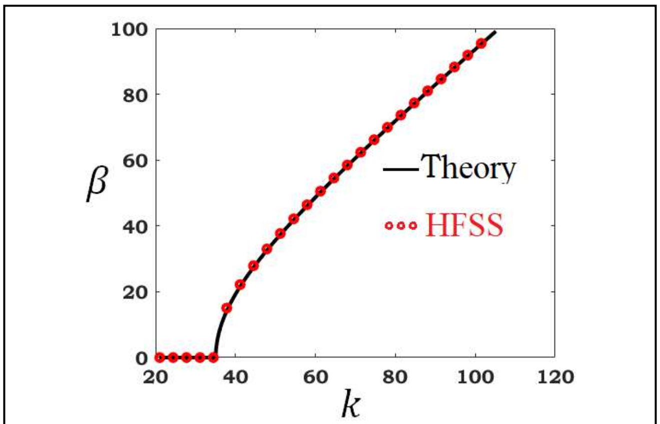

Fig. 4. k-β diagram of irregular pentagonal waveguide for $a = 36\ mm, b = 34\ mm$

8. Conclusion

An irregular pentagonal shape waveguide is investigated using conformal mapping technique (CMT) in this manuscript. This technique calculates cutoff- frequency (f_c) of this waveguide for the fundamental TE mode. In our theory, we can predict the lowest f_c with an accuracy of less than 2%. The internal field patterns are plotted in the original domain, i.e. (w-plane), using the inverse mapping technique. The relation between β (here, $k_z = \beta$) and k of the pentagonal waveguide is also studied with the help of the CMT. This theoretical approach can be extended to study other complex waveguides.

9. References

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