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A Study on the Impact of Kalman Filtering on CGGTTS formatted GPS data for Time Transfer

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Abstract

This paper applies a Kalman Filter on GPS data to study its efficacy in time transfer. This exercise has been carried out with the help a particular set of data as a case study to achieve the objective. This has been elaborated in this paper.

1. Introduction

All four constellations of Global Navigation Satellite System (GNSS) are almost fully operational. GNSS is also augmented by NavIC constellation particularly in India. Nevertheless, GPS signals are primarily used for the precise timing application. The special GNSS receivers for timing generate output in CGGTTS format [2]. In this format, it not only includes difference between local clock and received GPS time with applied error correction to the data but other useful relevant parameters for further processing. These data are directly utilised for comparing two participating clocks. Instead of directly using these data, one may pre-process these data by applying the Kalman filter technique This paper studies the utility of the Kalman filtering of the data for precise time transfer.

2. Kalman filtering

Kalman filtering [1] is a well-established technique widely used for the various applications. The application in this case involves one dimensional Kalman Filtering. For this type of timing data in CGGTTS format basic equations are rewritten as given in equations (1) to (5). The quantity for estimation at date (t+τ) is the time difference is
x(t+τ) =[Local Clock- GPS time] at (t+τ).
Let us define "x(t+τ/t)" is "the estimation of x at (t+τ) knowing t".
The estimation a priori of the time difference is

$$x(t+\tau/t) = x(t/t) + y_o \times \tau + v(t+\tau) \qquad (1)$$

where y_o is a constant frequency offset of the clock. v(t) is the white frequency noise which affects y_o and characterised by $\sigma^2(\tau)$. The constant frequency y_o can be estimated from earlier comparisons of the local clock and of GPS time with a more stable reference.
'p (t+τ/t)' is the prediction / estimate uncertainty variance is defined by the following equation

$$p(t+\tau/t) = p(t/t) + \sigma(\tau)^2\tau^2 \qquad (2)$$

The prediction / estimate uncertainty variance of p (t+τ/t) is updated by the following equation

$$p(t+\tau/t) = [p(t/t) + \sigma^2\tau^2] \times [1 - k(t+\tau) \qquad (3)$$

Kalman gain equation is governed by the relation

$$k(t+\tau) = (p(t/t) + \sigma(\tau)^2\tau^2)/(r(t+\tau) + p(t/t) + \sigma(\tau)^2\tau^2) \quad , \qquad (4)$$

'r(t+τ)' is the measurement uncertainty (variance) at (t+τ).

Finally, estimate of x $(t+\tau\,/\,t+\tau)$ may be obtained by

$$x(t+\tau/t+\tau) = [x(t/t) + y_o \times \tau] \times [1 - k(t+\tau)] + k(t+\tau) \times z(t+\tau) \qquad (5)$$

z$(t+\tau)$ is the measurement output at time $(t+\tau)$.

3. Implementation

The quantity for estimation here is (Local Clock- GPStime) termed 'REFSYS' in CGGTTS format. In this we have focussed on the data of gps-p3-cor data for particular lab from the BIPM site.
In this case, 'REFSYS' shown in 10th column and DSG shown in 12th column of the CGGTTS format are of interest in this analysis. The measurement z(t) taken at t, is characterized by the measurement error (variance) r(t). So, the REFSYS corresponds to z (t). DSG^2 represents r(t). (In CGGTTS format the root mean square of the individual data around the regression is named as DSG. As the variance is used in Kalman filter, so to use DSG as r(t), squaring of DSG is necessary.).

Let us assume for initialisation, $x(t_0/t_0) = z(t_0)$ = 'REFSYS' at t_0 of CGGTTS format and also $p(t_0/t_0) = r(t_0/t_0) = (DSG)^2$ at the start only.
$y_0(t)$ and $\sigma^2\tau^2$ have to be determined based on the observed data of the respective lab.

4. **Data analysis**

BIPM site publishes GPS data of a lab in three versions. One is the raw data as received from the receiver with name gps. BIPM process this data by the P3 correction for minimizing ionospheric error named as gps-p3. BIPM further process this data to reduce orbit and satellite clock error also names as gp-p3-cor. We concentrate on the third version of GPS data obtained from BIPM site for only two labs PTB and OP for the month of January 2022 spanning over 35 days between 59577 MJD to 59611 MJD for the case study. Duration of each data set is 780 s and each set repeats every 16 minutes. For each observation time 9 to 7 satellites were visible giving rise to simultaneously equal number of values of REFSYS. Kalman filter has been suitably applied to get the smoothed data for each observation.
Figure 1 and Figure 2 illustrate the values of Kalman processed data super imposed on the respective unprocessed GPS observation. On these figures the daily

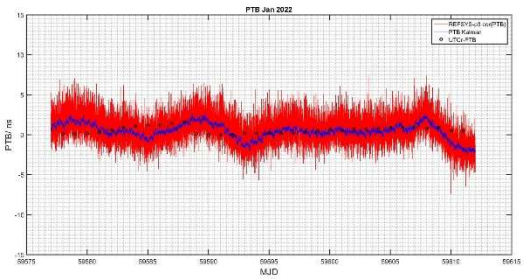

Figure 1. Output of Kalman estimation is superimposed on the raw data of PTB during January 2022.

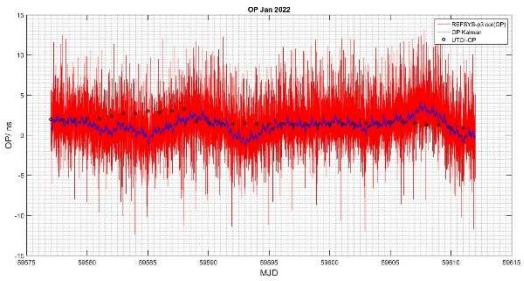

Figure 2. Output of Kalman estimation is superimposed on the raw data of OP during January 2022.

data of UTC-UTC(k) of the respective lab obtained from the 'rapid' publication of BIPM are also included. It is observed that Kalman processed data are quite close (within 1-2 ns) to the respective (UTC-UTC(k)) values.
It is clear from Figure 1 and Figure 2, the fluctuations in GPS data are slightly more for OP than those observed in PTB. The corresponding 1σ has been found to be 2.17 ns and 1.618 ns for OP and PTB respectively.

5. All-in-View Comparison

All-in-view (AV) is a standard technique that is used to compare the relative time offset between two timing labs [3],[4]. To use AV using Kalman filtering, Kalman estimated values for each observation time are averaged out to get final value of Kalman estimation for each observation time This is Kalman average of all-in-view satellites. Let us designate it as AV-Kalman(lab(k)). Raw values of REFSYS obtained for all visible satellites at a particular time for each observation time are also averaged to get average value of all-in-view, named as AV(lab(k)) for a particular observation time. Similar data processing is done for two labs PTB and OP. We have now two sets of the corresponding data – one is obtained directly using GPS data and the other one is found using Kalman estimated data. From these sets of data for the pair of labs, we find AV(PTB-OP) and AV-Kalman (PTB-OP). These two sets of data have been shown in Figure 3. It is clearly seen that AV-Kalman shows much reduced fluctuations (1σ = 0.575 ns) than those (1σ =1.345 ns) seen in AV. Closeness of these data to the superimposed of the daily values of (PTB-OP) obtained from the publication of BIPM is encouraging.

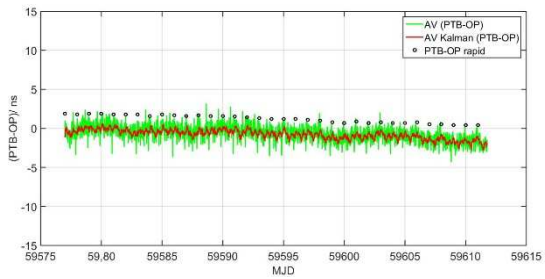

Figure 3. Relative performance of AV and AV-Kalman for PTB-OP. (PTB-OP) rapid is superimposed for comparison.

It may be noted that y-axes of Figures 1 to 3 are on the same range of scale for the visual assessment of relative performances without any confusion.
Frequency stability of AV and AV-Kalman has been studied. The outcome of this study has been shown in Figure 4.

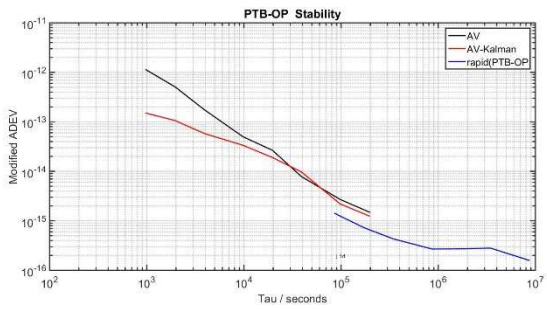

Figure 4. Frequency Stability of AV and AV-Kalman and (PTB-OP) rapid.

It interesting to note that the short-term stability of modified allan deviation of Kalman-AV is one order better

than that of AV. However, at larger tau, the frequency stability of both AV and Kalman AV tends to have almost same values. Rapid daily values UTC(PTB)-UTC(OP) repeat only at the interval of one day. So, Frequency stability of UTC(PTB)-UTC(OP) cannot be found for the tau of less than one day. So, the corresponding stability curve is shifted to the later part of tau in Figure 4. It may also be noted that at larger of values of tau, the stability of both the AV techniques gradually approaching to that of UTC(PTB)-UTC(OP). This would have been more evident, had this exercise been carried out for more longer period.

5. Conclusion

This paper introduces the AV-Kalman where the data are first denoised by Kalman filtering before applying AV technique to compare two participating clocks. It has been found that AV-Kalman technique reduces the fluctuations in the output considerably over those in AV. Through AV-Kalman, the short-term stability also improves by an order. In view of this, AV-Kalman may be used beneficially for time comparison between two clocks.
BIPM site has stopped publishing GPS data particularly for the lab whose PPP data are published. PTB being the pivot lab. both GPS data and PPP data of PTB are published. So, this exercise cannot be carried out by any lab unless the lab processes their own data without depending on the processing effort of BIPM.

6. References

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