Statistical Electromagnetic Theories and Applications, and Some Outstanding Problems

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Abstract – Statistical electromagnetic theories encompass two important areas of study. First, they are based on the fundamental studies of multiple scattering of waves. Theoretical formulations were developed by Tatarskii, Dyson, Bethe and Salpeter, and others, using techniques such as Feynman diagrams and radiative transfer. Mathematical and theoretical studies are still challenging today, requiring further development. Second, statistical wave theories have far-reaching practical applications in geophysical and biological media, including remote sensing, imaging, detection, communications, and signal processing. Therefore, these studies are directly applicable to the critical issues of today, including the environment and human health. Statistical electromagnetic theories challenge highly mathematical theoreticians and engineers interested in broad practical applications. This article attempts to summarize and discuss the key achievements in various areas of electromagnetic theories of random media and their applications, and point to some outstanding problems.

1. Introduction

Electromagnetic wave theories for deterministic media and boundaries have been extensively studied. In contrast, many natural geophysical and biological media vary randomly in space and time, which makes statistical descriptions and analysis essential [1–3]. Examples are remote sensing of geophysical media, imaging of biological media, and bio-optics and ultrasound imaging. These studies often require integration with signal processing [4]. Such applications have become increasingly important in satellite remote sensing and bioimaging and detection. However, these applications demand development and clear understanding of statistical multiple-scattering theories in random media. Understanding waves in random media is interdisciplinary, encompassing geophysics, optics, acoustics, and multiple-scattering theories used in astrophysics.

2. Statistical Wave Theory

Many natural and biological media vary randomly in space and time. How a wave interacts with random media and how we formulate these interactions mathematically are the central ideas of statistical wave theory. Though the waves vary randomly in space and time, there are well-defined theories underlining the random phenomena. Statistical wave theory aims at discovering and making use of these theories governing random phenomena for many practical and useful applications. As we deal with statistical wave theory, there are two fundamental equations: the Dyson equation for the average or the first moment and the Bethe–Salpeter equation for the second moment [3, 4].

As an example, consider the average or coherent Green's function $\langle G \rangle$ in a random medium with dielectric constant ε; it satisfies the integral equation

$$\langle G \rangle = G_0 + \int G_0 M \langle G \rangle dv, \tag{1}$$

where M is the mass operator. This is called the Dyson equation. The second moment $\langle G_1 G_2^* \rangle$ satisfies the Bethe–Salpeter equation:

$$\langle G_1 G_2^* \rangle = \langle G_1 \rangle \langle G_1^* \rangle + \int \langle G_1 \rangle \langle G_2^* \rangle I \langle G_1 G_2^* \rangle dv_1 dv_2, \tag{2}$$

where I is the intensity operator. (1) and (2) are basic equations.

Early studies of radio-wave scattering in the troposphere were conducted by Booker and Gordon [8]. Scattering by ionospheric turbulence has been studied extensively, including dispersion, the scintillation index, and synthetic-aperture radar imaging through the ionosphere. Atmospheric optics has been extensively studied, including strong fluctuation, thin-screen theory, and extended Huygens–Fresnel techniques. Also noted are the path-integral approach and the two-frequency mutual coherence function and time-domain solutions [1–4].

3. Applications

There are many applications involving detection and imaging in geophysical media and biological media. In order to understand interactions between electromagnetic waves and geophysical media, it is important to study multiple-scattering theory for satellite remote sensing, signal processing, imaging, detection, communications, and microwaves; turbulence in the ionosphere, atmosphere, and ocean; effects of particulate matter including fog, rain, smog, and snow; effects of terrain, vegetation, the ocean surface, underground, and underwater, including seismic coda waves; thermal emissions, soils, clouds, snow, and ice; and vegetation, wind, and soil moisture. Biological applications include bioelectromagnetics, bio-optics, bioultrasound, and imaging and detection of tissues and blood using optical, ultrasound, and electromagnetic modalities [4–8].

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4. Bio-Optics and Ultrasound in Tissues and Blood

In general, scattering in a random medium can be classified as single scattering, double scattering, multiple scattering, or diffusion. Optics in tissues or blood is dominated by diffusion, and absorption is small, while ultrasound in tissues is dominated by small scattering and large absorption. Therefore, in tissues an optical beam needs to be studied by the diffusion equation, whereas an ultrasound beam must be studied by the single-and-double-scattering approximation [8].

Among many applications of bio-optics, optical coherence tomography is well known. It makes use of an incident wave (low-coherence wave), with a coherent length of approximately 10 μm, and a Michelson interferometer to obtain a resolution on the order of the coherent length for imaging of the retina. Another example is passive radar using a low-coherence commercial FM (frequency modulation) radio [8].

There have been extensive studies on the use of ultrasound in blood and tissues [9, 10]. We need to consider the density and compressibility as well as the scattering amplitude and differential cross section per unit volume, and the packing factor is included to express the effect of high hematocrit.

5. Communications Through Random Media

Extensive work has been done on communications through various media. However, if the medium is random with deterministic boundaries, it is necessary to include diffraction and the mutual coherence function. For example, to study channel capacity it is necessary to include the signal-to-noise ratio, channel transfer matrix, and eigenvalues for waves in a random medium. To include diffraction effects, it is necessary to study the mutual coherence function including the deterministic boundary, such as the keyhole problem of current interest [8].

One of the important studies is the effect of random media on channel capacity. For a multiple-input, multiple-output propagation channel, the upper bound C_{up} is given by the transfer matrix, mutual coherence function, and eigenvalues [11]:

$$C \leq C_{\mathrm{up}} = \log_2\left[\det\left(\mathbf{I} + \left(\frac{\rho_t}{M\sigma^2}\langle \mathbf{T}\mathbf{T}' \rangle\right)\right)\right], \quad (3)$$

where $\mathbf{T}_{mn}$ is the channel transfer matrix

$$\begin{aligned}\mathbf{T}_{mn} &= F_{rn}F_{tm}g_{mn}\left(\vec{r}_n,\vec{r}_m\right)\\ &= \sum_i \log_2\left[\det\left(\mathbf{I} + \left(\frac{\rho_t}{M\sigma^2}\lambda_i\right)\right)\right],\end{aligned}$$

λ_i are the eigenvalues of $\langle \mathbf{T}\mathbf{T}' \rangle$,

$$\langle \mathbf{T}\mathbf{T}' \rangle_{nn'} = F^2 \sum_m \Gamma_{nn'},$$

and the mutual coherence function $\Gamma_{nn'} = \left\langle g_{nm} g^*_{mn'} \right\rangle$.

The mutual coherence function contains information about random media characteristics. This requires a study of waves propagating and scattering in a mixed medium that is random and deterministic, a problem that has not been completely solved yet.

6. Sommerfeld Dipole Problem on Rough Surfaces

The Sommerfeld dipole problem on conducting flat earth has been studied for over 100 years. However, detailed attention has not been paid to the problem when the surface is rough, since this requires statistical analyses and studies of coherent and incoherent fields and Dyson and Bethe–Salpeter equations for rough surfaces. The Sommerfeld problem with rough surfaces has not been studied completely except for perturbation solutions and approximate effects on the Sommerfeld pole. This includes the effects of rough surfaces and terrain, which have not been researched yet [8].

Related to this problem are low-grazing-angle imaging and detection over a rough ocean surface, including the correlation between forward and backward waves, and continuous-wave and pulse radar cross sections, requiring fourth-order moments [12]. The low-grazing-angle problem including polarization effects has been studied for a rough surface only for small root-mean-square heights.

7. Pulse Propagation Through Fog and Atmosphere or Over Rough Surface or Terrain, and the Study of Continuous-Wave and Pulse Radar Cross Section

Pulse propagation through random media is one of the problems of current interest. For example, a pulse radar on an autonomous car is used to detect and image objects in front of the vehicle or in surrounding areas. The received signal and correlations are used to sense the objects. Here, radar cross sections and imaging in the presence of random media and rough surfaces of both continuous waves and pulses are of great interest [4, 8].

The problem of a random medium with an object with a smooth surface can be solved using the approximate solution including time dependence. The short-pulse radar cross section requires a round-trip Green's function of fourth-order moments, including the correlation between the forward and backward propagations. The complete time-domain solution for radar cross section including a rough surface and moving targets has not been fully obtained [8].

8. Hard-Wall Imaging and Seeing Around Corners

In 1954, Sommerfeld wrote a classic text, *Optics*. In that text, he discussed *Poisson diffraction*, where waves from a source propagate around an object and form an image. A similar diffraction image occurs around the spherical earth; this is called *antipodal imaging*. In recent years, through-wall imaging has been studied, including the use of the waves through obstacles and walls. However, one recent imaging problem is to obtain images hidden behind obstacles [8, 13]. There has been much interest in these imaging techniques in recent years, under the names *hard-wall imaging* and, in optics and photonics, *seeing around a corner*. This technique appears to have attracted considerable attention. Hard-wall imaging has been studied using the geometrical theory of diffraction, time-reversal imaging, and intensity-multiplication techniques. Further work is needed for more complete study.

9. Multiphysics Including Random Media

An example of this problem is acoustic seismic waves in heterogenous random earth, called the *coda wave*. This study should include the effects of random media and rough surfaces on the propagation of Rayleigh surface waves.

Waves in homogeneous earth were extensively studied by Lamb, Cagniard, de Hoop, and Achenbach [8]. For heterogeneous earth Ishimaru and Sato, Fehler, and Maeda conducted studies showing that seismic coda waves can be approximately expressed as a wave train following the space-time homogeneous-earth solution. The study of coda waves requires further research [14]. One topic is the effect of random media on space-time surface waves. This is an important topic, since coda waves show the characteristics of the source and the medium and whether an earthquake is artificial or natural. This has not been fully investigated.

10. Mixing Formulas, Porous Media, and Percolation

Mixing formulas are well known and have been used to study the scattering and absorption of mixtures. In contrast, porous media are the opposite of mixtures, with particles that are voids. The basic ideas are similar, though, and the two can be treated with similar theoretical approaches [8].

Well-known formulas such as those of Maxwell Garnett, Rayleigh, Polder and van Santen, and Bruggeman have been used for porous media. However, more detailed theories and comparisons with experimental data are needed. Also needed is the study of percolation through porous media [15]. Wave characteristics in porous media also need further study, as well as the wavelength dependence of wave propagation in such media.

11. Time-Reversal Imaging and Signal Processing in Random Media

Extensive work has been reported on the use of time-reversal imaging, combining several techniques such as multiple-signal classification, decomposition of the time-reversal operator, and singular-value decomposition. Time-reversal imaging clarifies the super-resolution making use of correlation [8, 16, 17]. It should be noted that for random media, time-reversal techniques work as long as the random media are stationary in time.

12. Nonreciprocity in a Random Medium

Recently there have been extensive studies reported on nonreciprocity. Reciprocity from isotropic media was reported long ago as the *Lorentz reciprocity theorem*. Reciprocity theory was extended for anisotropic materials in a similar *modified Lorentz theorem*.

Waves in a random medium when correlations are included exhibit certain features of nonreciprocity, such as the case of the *shower curtain effect*. Further studies have also been reported for more general situations—for example, the correlation between forward and backward waves shows that the received power is different from that in the case without correlation [4]. One important example is the one-way transmission line and the thermodynamic paradox, which was clarified on the basis of an improperly posed problem [8].

13. Radiative Transfer

Radiation through a foggy atmosphere was studied by Schuster in 1903, and in 1950 Chandrasekhar published a definitive work on radiative transfer. Radiative transfer deals with the transport of energy and its heuristic development, and the addition of power holds rather than fields. It is equivalent to Boltzmann's kinetic theory of gases and to neutron transport theory [4, 8, 18]. It is applicable to geophysical imaging, detection, remote sensing, atmospheric and underwater visibility, bioimaging, atmospheres of planets, stars, and galaxies. It includes polarization and the Stokes vector formulation, and gives information about correlation and phase. Conservation of energy (a necessary condition) is satisfied, but the sufficient condition is not included. Many attempts have been made to develop a derivation of the radiative-transfer equation from Maxwell's equation. It was extended to space-time and two-frequency radiative transfer, and its relation to the Wigner distribution was clarified [8].

14. Radiative Transfer and Anderson Localization

In 1958, Anderson predicted the absence of diffusion and localization of electrons. Conductance of electrons is at the very heart of condensed-matter physics [19]. This includes the classical Drude model. The diffusion model and Anderson's localized electrons are based on multiple scattering in quantum mechanics. In 1984, attempts were made to see whether classical waves (optical) also exhibit (weak) Anderson localization. Since then, many studies have noted that Anderson localization is observed in different media and wave types, including clouds, fog, white paint, biological media, microwaves, acoustics, seismic waves, complex materials, enhanced backscattering, and coherent backscattering. Enhanced backscattering from particles was discovered experimentally by Kuga and Ishimaru (1984) using an optical experiment and theoretically explained by Tsang and Ishimaru (1984) using a Feynman diagram [8, 19, 20].

15. Enhanced Radar Cross Section Through Random Medium

It may be expected that radar cross section (RCS) is reduced through random media. However, it has been experimentally observed that RCS is actually increased through turbulence. The normalized radar cross section shows that RCS with turbulence is twice the RCS without turbulence, and the scintillation index is 5. This is called the *double passage effect* and is related to Anderson localization [8, 20].

16. Conclusion

Statistical electromagnetic theories present very challenging theoretical and mathematical modeling and techniques, as well as the possibility to develop new theories and techniques to address problems in broad areas of electromagnetics. Their applications include areas of remote sensing, imaging, and detection in geophysical and biological media, which are fundamental to environmental and health concerns, two of the most critical issues of this century.

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18. References

1. V. I. Tatarskii, *The Effects of the Turbulent Atmosphere on Wave Propagation*, Jerusalem, Israel Program for Scientific Translations, 1971.
2. V. I. Tatarskii, *Wave Propagation in a Turbulent Medium*, New York, McGraw-Hill, 1961.
3. U. Frisch, “Wave Propagation in Random Media, ” in A. T. Bharucha-Reid (ed.), *Probabilistic Methods in Applied Mathematics, Vol. 1*, New York, Academic Press, 1968, Chapter 2.
4. A. Ishimaru, *Wave Propagation and Scattering in Random Media*, New York, IEEE Press and Oxford University Press, 1997.
5. J. A. Kong, *Electromagnetic Wave Theory*, New York, Wiley, 1986.
6. L. Tsang, J. A. Kong, and R. T. Shin, *Theory of Microwave Remote Sensing*, New York, Wiley-Interscience, 1985.
7. F. T. Ulaby, R. K. Moore, and A. K. Fung, *Microwave Remote Sensing: Active and Passive*, Vols. 1, 2, and 3, London, Addison-Wesley, 1981-1986.
8. A. Ishimaru, *Electromagnetic Wave Propagation, Radiation, and Scattering: From Fundamentals to Applications*, 2nd ed., Hoboken, NJ, IEEE Press and Wiley, 2017.
9. J. C. Lin, *Electromagnetic Fields in Biological Systems*, Boca Raton, FL, CRC Press, 2012.
10. K. K. Shung and G. A. Thieme (eds.), *Ultrasonic Scattering in Biological Tissues*, Boca Raton, FL, CRC Press, 1993.
11. A. Ishimaru, S. Jaruwatanadilok, J. A. Ritcey, and Y. Kuga, “A MIMO Propagation Channel Model in a Random Medium,” *IEEE Transactions on Antennas and Propagation*, **58**, 1, January 2010, pp. 178–186.
12. A. Ishimaru, J. D. Rockway, and S.-W. Lee, “Sommerfeld and Zenneck Wave Propagation for a Finitely Conducting One-Dimensional Rough Surface,” *IEEE Transactions on Antennas and Propagation*, **48**, 9, September 2000, pp. 1475–1484.
13. A. Ishimaru, C. Zhang, and Y. Kuga, “Hard Wall Imaging of Objects Hidden by Non-Penetrating Obstacles Using Modified Time Reversal Technique,” *IEEE Transactions on Antennas and Propagation*, **62**, 7, July 2014, pp. 3645–3651.
14. H. Sato, M. C. Fehler, and T. Maeda, *Seismic Wave Propagation and Scattering in the Heterogeneous Earth*, 2nd ed., Heidelberg, Germany, Springer, 2012.
15. D. Stauffer and A. Aharony, *Introduction to Percolation Theory*, 2nd ed., London, Taylor & Francis, 1991.
16. A. J. Devaney, *Mathematical Foundations of Imaging, Tomography and Wavefield Inversion*, Cambridge, UK, Cambridge University Press, 2012.
17. M. Fink, “Time Reversed Acoustics,” *Physics Today*, **50**, 3, March 1997, pp. 34–40.
18. S. Chandrasekhar, *Radiative Transfer*, New York, Dover, 1960 (reprint).
19. P. W. Anderson, “Absence of Diffusion in Certain Random Lattices,” *Physical Review*, **109**, 5, March 1958, pp. 1492–1505.
20. A. Ishimaru, “Backscattering Enhancement: From Radar Cross Sections to Electron and Light Localizations to Rough Surface Scattering,” *IEEE Antennas and Propagation Magazine*, **33**, 5, October 1991, pp. 7–11.