Simulator for the transionospheric channel including strong scintillations

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A GNSS satellite to ground channel model including strong ionospheric scintillations

A transionospheric propagation model has also been developed which can calculate frequency spectra (power spectra) of the phase and level (log-amplitude) fluctuations in a transionospheric channel of propagation containing time-varying electron density irregularities which produce scintillations.

Relevant codes have been created to calculate time correlation functions of the phase and log-amplitude fluctuations for real 3D models of the background ionosphere and the anisotropic inverse power law spatial spectrum of fluctuations of the electron density of the ionosphere.

Calculated power spectra of these processes can then be employed to produce random time sequences of the log-amplitude and phase of the field.
Propagation from a satellite to the Earth's surface is calculated in two steps:

i) spatial spectra of the field phase and amplitude are calculated by the complex phase method for the points of observations below the ionosphere. The spectra are employed to generate a random screen below the ionosphere;

ii) the propagation problem for a random screen is rigorously solved to obtain the field’s statistical moments and time series on the Earth's surface.
This hybrid technique allows:

- the case of strong scintillation of the field amplitude to also be considered;

- Achievement of the higher accuracy of numerical modelling of realistic cases of propagation compared with the case of straightforward purely numerical multiple phase screen calculations.

The complex amplitude of the field passed through the ionosphere is represented as follows

\[ E(r, \omega , t) = E_0(r, \omega )R(r, \omega , t) \]

undisturbed field  __________  random factor
Random factor $R(\mathbf{r}, \omega, t)$ is treated in terms of the complex phase

\[ R = e^\psi, \quad \psi = \chi + iS. \]

Complex phase phase log-amplitude

To formulate a random screen under the ionosphere, the two-dimensional spatial spectra for phase and log-amplitude are produced, which are then employed to generate two-dimensional realisations of $\chi$ and $S$. 
Spectra of phase, log-amplitude and their cross-correlation

\[ B_S(\kappa_n, \kappa_\tau) = \frac{k^2}{4} \int_0^{S_0} ds \frac{B_\varepsilon(0, \kappa_n, \kappa_\tau, s)}{\varepsilon_0(s)} \cos^2 \left\{ \frac{1}{2k} \left[ \kappa_n^2 D_n(s) + \kappa_\tau^2 D_\tau(s) \right] \right\} \]

\[ B_\chi(\kappa_n, \kappa_\tau) = \frac{k^2}{4} \int_0^{S_0} ds \frac{B_\varepsilon(0, \kappa_n, \kappa_\tau, s)}{\varepsilon_0(s)} \sin^2 \left\{ \frac{1}{2k} \left[ \kappa_n^2 D_n(s) + \kappa_\tau^2 D_\tau(s) \right] \right\} \]

\[ B_{S\chi}(\kappa_n, \kappa_\tau) = -\frac{k^2}{8} \int_0^{S_0} ds \frac{B_\varepsilon(0, \kappa_n, \kappa_\tau, s)}{\varepsilon_0(s)} \sin \left\{ \frac{1}{k} \left[ \kappa_n^2 D_n(s) + \kappa_\tau^2 D_\tau(s) \right] \right\} \]
In the numerical simulation, the anisotropic inverse power law spatial spectrum of fluctuations of the ionospheric electron density is employed

\[ B_\varepsilon (\vec{k}, s) = C_N^2 \left[ 1 - \varepsilon_0(s) \right]^2 \sigma^2_N \left( 1 + \frac{k_{tg}^2}{K_{tg}^2} + \frac{\vec{k}_{tr}^2}{K_{tr}^2} \right)^{-p/2} \]

- \( C_N^2 \) - normalisation coefficient
- \( \varepsilon_0(s) \) - distribution of the dielectric permittivity of the background ionosphere along the reference ray
- \( \sigma^2_N \) - variance of the fractional electron density fluctuations
Realisations of phase and amplitude on the random screen $f = 1575$ MHz

The random screen which is generated below ionosphere is not an equivalent phase screen, but the screen generated on the basis of solving the propagation problem through the fluctuating ionosphere specified by its electron density profile and a given model of fluctuation spectra.
The random spectrum $\tilde{E}(0, \kappa, t)$ of the field on the screen is then transferred to the level of the Earth’s surface employing the following relationship of the theory of a random screen

$$\tilde{E}(z, \kappa, t) = e^{ikz} \tilde{E}(0, \kappa, t) \exp\left(-\frac{i\kappa^2z}{2k}\right)$$

**Examples of model outputs**

**Input parameters:**
NeQuick profile, low-latitude ionosphere, TEC = 69 TECU. spectral index $p=3.7$.
Variance of fluctuations $\sigma^2_N = 10^{-2}$.
cross-field outer scale 10 km, aspect ratio $a = 5$.
Path of propagation: elevation angle of $45^0$, azimuth $45^0$.
The effective velocity of the horizontal frozen drift 300 $m/s$.
Frequencies $f = 1575$ MHz and $500$ MHz.
Time realisations of phase and amplitude on the Earth’s surface
\( f = 1575 \text{ MHz} \)
Spectra of phase and amplitude fluctuations

Probability density function of amplitude fluctuations

\[ f = 1575 \text{ MHz} \]
Rate of phase changes, \( f = 1575 \text{ MHz} \)

\[ \sigma^2 = \left\langle \left( \frac{\partial S}{\partial t} \right)^2 \right\rangle \]
Realisations of phase and amplitude on the Earth’s surface.

\[ f = 500 \text{ MHz} \]
Time realisations of phase and amplitude on the Earth’s surface 
\(f=500\ \text{MHz}\)
Spectra of phase and amplitude fluctuations

Probability density function of amplitude fluctuations

$f=500\ \text{MHz}$
Rate of phase changes, $f = 500$ MHz
Rate of change of phase for strong scintillations

The rate of change of phase is obtained by numerical differentiation of the time series of phase. For 45% fluctuations, this can be in excess of 20 rad/s (~ 3Hz) as shown in the figure below left. This is less than the typical GPS phase lock bandwidth of 10 Hz.

However, the cross-correlation function of the absolute values of the phase changes and log-amplitude variations were also determined for strong scintillation conditions and the correlation is seen to have a clear minimum at the zero time difference.
Correlation between deep amplitude fades and fast phase changes

This minimum in the correlation indicates occurrences of fast change of phase are associated with deep fading of the field (for these large fluctuations) so both effects need to be considered together to properly assess phase lock loss conditions.

A similar calculations was performed for the same conditions of propagation but for a smaller value of the variance of the fractional electron density of 0.01 (10%). In this case there was practically no correlation between phase changes and fading of the field.
Conclusion

The presented technique is capable of producing statistical characteristics and of simulating time realisations of the field (including regime of strong amplitude fluctuations) for a wide range of the input parameters, viz:

- co-ordinates of the satellite and point of observation
- slant electron density profile along a given path
- zenith angle of a satellite
- magnetic azimuth of the plane of propagation
- magnetic field dip angle at the pierce point
- the following parameters of the random irregularities:
  - spectral index
  - outer scale across the geomagnetic field
  - aspect ratio
  - variance of the fractional electron density fluctuations
  - effective velocity of the drift