APPENDIX D. GROUP INDEX CALCULATIONS WITHIN POLAN

D.1 Calculation of the virtual height coefficients - The subroutine COEFIC

D.1.1. The equations used in COEFIC.

POLAN normally assumes (as in Section 4.2) that real-height segments can be represented by a polynomial of the form

$$HR - HA = \sum_{j=1}^{NT} q_j.(FR-FA)^j = \Sigma\, q_j.a_j. \qquad (D1)$$

HA is the height at the frequency FA (the "origin" for the real-height section to be calculated), and HR is the real height at the plasma frequency FR. The corresponding virtual-height expression is

$$h'' - HA = \sum_{j=1}^{NT} q_j.b_j \qquad (D2)$$

where

$$b_j = j \int_{FA}^{FR} \mu'.(FN - FA)^{j-1}\, dFN. \qquad (D3)$$

h" is the virtual height, at the frequency FR, after group retardation due to ionisation below the origin (i.e. with FN < FA) has been subtracted. h" is called the reduced virtual height. The polynomial analysis consists basically in setting up equations from (D2), at each of the virtual height frequencies involved in a given real-height step; adding further equations corresponding to known real heights, from (D1); and solving the resulting set of simultaneous equations to obtain the real-height coefficients q_j.

The subroutine COEFIC is used to calculate the real-height coefficients a_j and the virtual height coefficients b_j, at each frequency involved in the analysis. For maximum accuracy we use $(\mu'-1)$ rather than μ', and integrate with respect to a variable T defined by $T^2 = 1 - (FN/FR)^2$. This gives a well-behaved integrand near reflection, where μ' becomes infinite. Thus the virtual height coefficients are calculated as

$$b_j = a_j + d_j \qquad (D4)$$

where

$$d_j = j \int (\mu'-1).(FN-FA)^{j-1}\, dFN$$

$$= j.FR^2 \int [(\mu'-1).T/FN].(FN-FA)^{j-1}\, dT \qquad (D5)$$

For a wave of frequency FR the integration range in (D5) is from T = 0 (the reflection height) to $TA = (1 - FA^2/FR^2)^{.5}$. Using an n-point Gaussian integration we must evaluate the integrand at n values of T within this range. If T_r represents these values, and W_r the corresponding Gaussian weights, we have

$$d_j = TA.j.FR^2 \sum_{r=1}^{n} G_r.W_r.(FN_r-FA)^{j-1} \qquad (D6)$$

where $G_r = (\mu'_r-1)T_r/FN_r$

and $FN_r = FR.(1-T_r^2)^{.5}$.

All the required coefficients d_j are obtained from the same n values of G_r, so that increasing the number of terms used in the polynomial real-height expansion (D1) does not increase the number of group index calculations required. Accurate values of $(\mu'-1).T$ are obtained from the subroutine GIND (Appendix D.3). They are calculated directly from the values of wave frequency FR and the value of T_r, to avoid the small difference errors involved in calculating FN_r from T_r.

Gaussian coefficients for both 5- and 12- point integration are contained in DATA statements in the subroutine COEFIC. They are stored sequentially so that an iteration loop can also use 17-point integration when required (as at high dip angles; Appendix B.3). With 17-point integration, calculations pause after the first 5 terms while adjustments are made to the integration limits and to the gyrofrequency used.

Start calculations incorporating X-ray data use 5-point integration for the linear slab section, with a gyrofrequency specified by the height FHHT. For the polynomial real-height section a 12-point integration is employed, on the first iteration. If this reveals a "difficult" profile (as determined by the tests (1) of Appendix C.6), subsequent iteration uses 17-point integration. This is split into

(a) a 5-point integral, from 0.6fmin up to the calculated boundary point between the underlying and reflection regions, using the value of FH at FHHT; and

(b) a 12-point integral from this boundary point to the reflection point for each ray, using FH at the reflection height of the first X-ray.

D.1.2. The operation of COEFIC.

Parameters passed from POLAN through the COMMON statement are:

NR = the total number of real heights to be fitted, (in addition to the origin at FA, HA).

NL = the number of real heights preceding the origin (= 0 or 1).

MT = the number of terms in the polynomial real-height expression, excluding any constant.

JM = the total number of terms, including any constant. Thus JM = MT+1 for start or valley calculations, when the final result will alter the real height at the frequency FA.

LK = the index of the last real height used for removing the effect of group retardation in underlying ionisation. LK is used within COEFIC only to show when the current step is a start calculation. LK = 1 for a normal start calculation, while values of 0 or -1 are used to indicate a polynomial or slab start using X rays. For normal coefficient calculations (which have MV > 0), we have LK = KR.

KR = the index of the real-height origin in the frequency, height arrays. Thus FA = FV(KR) and HA = HT(KR).

KRM = the index of the highest previously-calculated real height.

KV = the index of the virtual height corresponding to the origin (the frequency FA). This is greater than KR by the amount the virtual-height data was initially moved up in the arrays FV, HT - modified by deletion of X ray data and insertion of extra real heights in start or valley regions.

MODE specifies the type of analysis, from linear laminations (MODE = 1) to a single polynomial fitting all data (MODE = 10). The value of MODE is increased by 10 (giving values from 11 to 20) when maximum integration accuracy is required.

MOD is the value of MODE reduced to the range 1 to 10, for start or restart (after a peak) calculations; or the value of MODE increased to the range 11 to 20 at other times.

FA,HA give the actual origin (or the previous origin for reduction calculations, as in (C) below).

DIP and HS give the magnetic dip angle and an initial gyrofrequency height. Actual values of the gyrofrequency, at any height, are obtained from the subroutine GIND.

FC, SH give the critical frequency and the scale height of any relevant peak; otherwise FC = 0.

PARHT is the thickness of an above-peak parabolic section, extending to a valley depth VDEPTH.

XWAT gives the relative weighting for X ray data; this is normally 1 but may be halved by POLAN.

COEFIC is called with arguments MV, FV and HT. When the input parameter MV is positive it gives the number of frequencies at which virtual-height equations are required. COEFIC then carries out the operations described below. Labels CC1, CC2 etc. correspond to comments in the program listing. Steps which are relevant only when MV is negative, and COEFIC serves a different purpose (as described in Appendix D.2) are marked by **.

CC1 --- INITIALISATION

IF MODE > 8, OR the calculation includes X rays,
THEN use 12-point integration.
ELSE use 5-point integration.

** IF MV < 0 and X rays are not involved, use 5-point integration.

SET NF = MV = the total number of frequencies to be processed, from FV(KV+1) to FV(KV+NF).

SET FW = FV(KV+NF+1) to give the upper frequency at which the weighting of virtual-height equations is tapered to zero.

SET DW = the range (below FW) over which the weight increases to 1.0; this range includes all frequencies used in the calculation for which final real heights are NOT calculated in this step.

SET DA = the lower range over which weights increase, from zero (at FA) to 1.0 at the highest known real height.

IF LK < 0 (a slab start calculation), and MV > 0,
THEN calculate quantities for optimum estimation of the mean gyrofrequency height FHHT.

---------- **THE MAIN (FREQUENCY) LOOP IN COEFIC**

All following sections of COEFIC are repeated for each of the NF data frequencies. The index KVI accesses the required frequencies and virtual heights, from the data arrays HT and FV, by taking in successive loops the values (KV-1), KV+1, KV+2, . . KV+MV. The initial (bracketed) term is included only at NL = 1, when one real height is fitted below the origin (FA,HA).

CC2--- **FREQUENCY RANGE AND WEIGHTS**

SET The Virtual-height Index KVI, and the position I = KVI-KV to store the coefficients B(I,J).

SET F = FV(KVI), HV = HT(KVI).

IF F > 0 THEN SET FR = F (= the plasma frequency at reflection).
ELSE SET FR = SQRT[F(F + FH)].

IF KVI > KV THEN SET WREAL = 20 (The weight for real-height equations).
ELSE SET WREAL = 10. (Reduce the weight at FR < FA).

IF KVI > KV+NR THEN SET WREAL = 0. (Clear unused areas of array B).

IF F > 0 THEN SET WVIRT = 1. (The weight for virtual-height equations.)
ELSE SET WVIRT = XWAT. (The weight for X-ray data.)

SET IREAL = NF + NL + I (The row to store the real-height equation, following NF virtual-height equations.)

CC3--- **INTEGRATION LIMITS,** in terms of the variable T = SQRT(1. - $(FN/FR)^2$).

SET TB = SQRT(1 - $(FA/FR)^2$). The lower limit of integration.
SET TA = 0 The upper limit of integration.

** IF MV < 0, THEN IF PARHT = 0 SET TA = SQRT(1. - $[FV(KV)/FR]^2$)
** ELSE SET TA = SQRT(1. - $[FV(KV-1)/FR]^2$).
** GO TO CC4.

IF The Calculation includes X rays,
THEN set TC = the value of T at FN = FA + 0.1(FR-FA),
and use 17-point integration (5-point from TC to TB; + 12-point from TA to TC).
ELSE set TC = 0.39 - 0.05/cos(.016*/dIP) where DIP is in degrees.

IF TB > 1.2TC, and DIP > 60 (or 70) degrees at MODE < 8 (or > 7),
THEN use 17-point integration.

IF using 17-point integration, set TA = TC (the start of the 5-point integral).
SET TD = TB - TA (the size of the first integration interval).

CC4--- **Retardation in a START/VALLEY OR PEAK REGION,** using 12-point integration.

IF LK > 0 (not an X ray start calculation); AND PARHT = 0 (not a valley calculation)
THEN GO TO CC5.
ELSE (a valley calculation) set DEPAR = integral of GIND.T.DT for a parabolic section of unit thickness from the peak (FN = FA) to the valley bottom (FN = FA - VDEPTH).

SET DELIN = integral of (GIND-1).T.DT for FN increasing linearly from FA-VDEPTH to FA.

CC5--- **STORE** individual, weighted values of GROUP INDEX (GIND) and frequency offset.

C5.1 FOR Each Gaussian integration point T_r, in the range TA to TA + TD
(with r = 1 to 5, or r = 1 to 12):
STORE the corresponding plasma-frequency interval FN_r - FA as FNR_r;

STORE the value of $[(GIND_r - 1).T_r.W_r/FN_r].TD.FR^2$ in the array $GAUSS_r$.

C5.2 IF 17-point integration is being used, set TD = TA, TA = 0. and repeat C5.1 for the second (12-point) integration section.

CC6--- **CALCULATE AND STORE THE COEFFICIENTS B(I,J)**
for the Virtual and Real-height Equations, at the Frequency F_i.

This section is repeated JM times, with successive values of J = 1, 2, ...JM.

SET RH = $(FR - FA)^J$ giving the real-height coefficient A(I,J).

SET SUM = 0 (integral over the linear Start or Valley section).

IF J > MT THEN set DPEAK = DEPAR*PARHT (group integral for parabolic peak).
ELSE set DPEAK = 0.

IF J = MT and LK < 0 (a slab start calculation)
THEN set RH = 0 (no real-height component from underlying slab),
SUM = DELIN (group retardation in slab).

IF J = MT+1 set RH = 1.

IF J = MT+1 and LK > 0 (a **Valley** calculation)
THEN set SUM = Group retardation for valley base VB + linear step (1-VB).

FOR Normal **Polynomial Terms** ($J \leq MT$ at $LK \geq 0$, or $J < MT$ at $LK < 0$)
set SUM = J Σ $GAUSS_r$ (giving the group delay integral for B(I,J)),
$GAUSS_r = GAUSS_r.FNR_r$ (in preparation for the next term).

STORE B(I,J) = (SUM+RH)*WVIRT (the virtual-height coefficient).
STORE B(IREAL,J) = RH*WREAL (the real-height coefficient).

** IF MV < 0, replace HT(KVI) by HT(KVI)-Q(J)*SUM-DPEAK (subtract the group delay).

CC7--- **THE RIGHT HAND SIDE OF THE EQUATIONS.**

IF I > 0 THEN IF DPEAK = 0 set B(I,JM+1) = (HV-HA).WVIRT (no valley)
ELSE set B(I,JM+1) = (HV-HA-DPEAK).WVIRT (a valley calculation).

SET B(IREAL,JM+1) = (HT(KVI-KVR) - HA). WREAL (right hand side of real-height equation).

IF I = MV THEN RETURN from COEFIC.
ELSE increment KVI and I, and repeat CC2 to CC7.

D.2 Delay in the underlying ionisation - The subroutine REDUCE

Each step in the real-height calculation has determined a new section of the profile. Part of this section, for plasma frequencies FN extending from FA up to some top value (FT say), is retained as a final result. The group retardation due to this part must be subtracted from the virtual heights at higher frequencies. This "reduction" is carried out by the subroutine REDUCE and accounts for most of the time required by a full profile analysis. To minimise this time, the calculations in REDUCE use two different procedures.

D.2.1. Polynomial reduction, at low frequencies (section C2.A of REDUCE).

For frequencies up to some maximum value FRED, the group retardation is calculated using the full polynomial expression for the previous real-height section. FRED is normally about 0.6 MHz above the highest frequency to be used in the next stage of the analysis. (Slightly lower values of FRED are used with the simpler modes of analysis, to shorten the calculations.) The group retardation is calculated by COEFIC, using 5-point Gaussian integration of $\mu'-1$ over the required range of plasma frequencies. If h' is the previously-stored virtual height at some frequency, this is replaced by the new reduced height

$$h'' = h' - \sum_{j=1}^{NT} d_j . q_j \qquad (D7)$$

where d_j is defined in equations (D1) to (D6) above. The upper limit of integration in (D3) is changed from the wave frequency FR to the maximum plasma frequency (FT = FV(KV)) in the real-height section. Apart from this change, the reduction calculations proceed as described in D.1 above.

COEFIC is called from REDUCE at any early stage in POLAN, with an argument MV which indicates the number of frequencies at which virtual heights are to be reduced. The parameters MT, JM, LK (which indicates a slab or polynomial start, at LK = -1 or 0), FC, SH, PARHT (the thickness of a calculated topside peak section), and VDEPTH still retain the values associated with the previous real-height calculation step. Only the values KR and KV have been updated, to give the origin for the next step.

Group-delay integrals are required at the frequencies FV(KV+1) to FV(KV+NF), where NF = MV. Integrals are evaluated for plasma frequencies from FA (the starting frequency of the previous real-height polynomial) to FV(KV). A few additional tests marked by ** in D.1.2 speed the calculation by using only a five-point integration (except for the longer X-ray start section, when 12 points are used). The virtual-height coefficients d_j are calculated in the normal way. One additional statement, marked ** at the end of CC6, subtracts the corresponding group-delay term $d_j.q_j$ from the virtual height HT(KVI).

Thus at the end of COEFIC the effect of group retardation in the real-height section from the previous origin FA to the new origin FV(KV) has been accurately removed from all virtual heights at frequencies FV(KV+1) to FV(KV+MV).

D.2.2. Quadratic reduction at higher frequencies.

(i) For linear-in-FN slabs.

At wave frequencies F greater than FRED an approximate reduction is used, assuming some simplified form for the profile shape between scaled frequencies. For a fixed magnetic field the group refractive index μ' is a function of F and of the plasma frequency FN. At F >> FN the group retardation in an interval FN = F1 to FN = F2 is commonly determined as

$$R = \int (\mu'-1).dh = (\mu^*-1).\Delta h \qquad (D8)$$

where

$$\mu^* = 1/2\ (\mu'(F,F1) + \mu'(F,F2)) \qquad (D9)$$

This corresponds to trapezoidal integration. When a given frequency is reduced by the delay in successive intervals in FN, the top limit of the first integral is the bottom limit of the next. Only one new calculation of μ' is therefore required for each interval, if the basic reduction cycle calculates the retardation in successive real-height segments h0 to h1, h1 to h2, ... at a given frequency F. A disadvantage of this ordering is that, to make full allowance for the variation of gyrofrequency FB with height, the value of FB should be rescaled at each calculation of μ'.

The accuracy of the above procedure can be improved by a factor of about two if we replace (D9) by

$$\mu^* = \mu'(F,FNA) \qquad (D10)$$

where $FNA^2 = (F1^2 + F2^2)/2$. The improvement occurs because the integral of a parabolic expression is represented twice as accurately by the value at the midpoint as by the mean of the end-point values. Use of (D10) also removes the restriction that (to avoid calculating two new values of μ' for each interval) all calculations for a given frequency must be performed together. We can now take each newly-calculated real-height segment and correct all higher frequencies for the delay in this segment, using the (fixed) values of FNA and FB appropriate to the centre of the new segment. This avoids the re-scaling of FB and corresponding adjustments to the constants FBsinI, FBcosIcotI, ... ; calculations which add nearly 50% to the total time required for a group index calculation.

For frequencies greater than 1.2FN the integral in (D8) depends primarily on the total electron content over the height range Δh (Titheridge, 1959c). For this reason (D10) uses the RMS plasma frequency FNA to calculate μ^*. (D8) is then correct for a linear-in-N real-height interval Δh. Within POLAN, however, height h is expressed as a function of FN. The starting slab used with X ray calculations, and the top section of a valley, are linear in FN. While the choice between linear-in-N and linear-in-FN has little effect on the results, it is important that the same representation be used throughout. For a section $h = a + b.FN$ extending from F1 to F2 the total electron content is proportional to

$$I = \int FN^2.dh = b \int FN^2.dFN = FNA^2.(h2-h1) \qquad (D11)$$

where

$$FNA^2 = (F2^2 + F2.F1 + F1^2)/3 \qquad (D12)$$

This value of FNA then gives a group retardation (D8) corresponding to the correct total electron content.

(ii) For curved real-height sections (section C2.B of REDUCE).

POLAN calculates each real-height section as a polynomial in FN. A correction for profile curvature can then be made, using the known values of the gradient $G = dh/dFN$ at the ends of the interval. For this purpose we represent the real-height section from F1 to F2 by

$$h = a + b.FN + 1/2\ c.FN^2.$$

Using Δ to denote the change in any quantity from F1 to F2 this gives

$$\Delta G = c.\Delta FN$$

and

$$\Delta h = b.\Delta FN + 1/2\ c(F2 + F1)\Delta FN = b.\Delta FN + 1/2(F2 + F1)\Delta G.$$

Equation (D11) then becomes

$$I = b \int FN^2.dFN + c \int FN^3.dFN = 1/3\ b.\Delta(FN^3) + 1/4\ c.\Delta(FN^4)$$

$$= (\Delta h - 1/2(F2+F1).\Delta G).\Delta(FN^3)/(3\Delta FN) + \Delta G.\Delta(FN^4)/(4\Delta FN)$$

$$= \Delta h.FNA^2 + \Delta G.\Delta FN.\Delta(FN^2)/12 \qquad (D13)$$

or

$$I = DH.FNA^2$$

where

$$DH = (h_2 - h_1) + (G2-G1)(F2+F1)(F2-F1)^2/(12*FNA^2). \qquad (D14)$$

Calculations are therefore corrected for curvature of the real-height segments by assuming that the group retardation is

$$R = (\mu'(F,FNA) -1).DH \qquad (D15)$$

where FNA^2 is the effective mean value of FN^2, from (D12), and DH is a corrected height interval from (D14). The gradients G1, G2 at the end-points of each interval are normally obtained during a real-height analysis, so the only additional work required is evaluation of the final term in (D14). A negligible increase in computing time is involved, since the values of FNA and DH are independent of the wave frequency; they are calculated only once for a given real-height segment. With the corresponding value of gyrofrequency FB set in GIND, equation (D15) is then used to reduce the virtual heights at all higher frequencies.

The result gives a close approximation to the true group retardation for wave frequencies greater than 1.2F2. It is effectively an application of the Euler-Maclaurin formula, which shows that an n-interval trapezoidal integration of y(x) from x_1 to x_2 is corrected exactly for changes in the first and second derivatives by a term $(g_2-g_1)(x_2-x_1)/12n$ where $g = dy/dx$. In the present case we have derived a similar expression (D13) to correct the integral I for changes in the first derivative. The integrand and the definition of G differ somewhat from the Euler-Maclaurin form, so results will not be fully corrected for a changing second derivative. The differences in form tend to zero at $f >> F2$, when the use of (D14) corrects accurately for cubic profile segments.

D.3 Calculation of the group refractive index - The subroutine GIND

The speed of the POLAN analysis depends primarily on the speed of the group index calculation. The attainable accuracy can also be limited, under some conditions, by small-difference errors in the calculations of μ'. The following formulation completely avoids small difference errors for both the ordinary and extraordinary rays, for the full range of dip angles (I = 0 to 90°) and for plasma densities FN from 0 to the reflection point.

For vertical propagation in the upper atmosphere (and ignoring the effect of collisions) the Appleton-Hartree equation for the phase refractive index μ may be written

$$\mu^2 = 1 - X.F/D \qquad \text{(D16)}$$

where

$$X = FN^2/F^2, \qquad D = F + E \qquad \text{(D16a)}$$

$$G = B/F(1-X), \qquad E = A[(1+G^2)^{.5} - G] \qquad \text{(D16b)}$$

$$A = FH\sin I \quad \text{and} \quad B = .5FH\cos I\cot I. \qquad \text{(D16c)}$$

FN is the plasma frequency, FH the gyrofrequency and I the magnetic dip angle. When the wave frequency F is positive, (D16) gives the value of μ for the ordinary ray. Making F negative changes only the sign of the square-root term, when the equations are combined, giving the extraordinary ray result. This convenient form has been retained in the group refractive index equations (below) by suitable placement of the terms involving F. Hence throughout POLAN we adopt the convention that extraordinary rays are indicated by a negative value for the wave frequency F.

GIND calculates the refractive indices as a function of the wave frequency F and the parameter T defined by

$$T^2 = 1 - FN^2/FR^2. \qquad \text{(D17)}$$

FR is the plasma frequency at reflection, so that FR = F for the ordinary ray and $FR^2 = F(F+FH)$ for the extraordinary ray (which has F negative). T is the independent variable in the group index integrations in COEFIC. Use of T as a parameter, rather than FN, speeds the calculations and avoids the small-difference errors which arise (near the reflection point) if T must be calculated from FN within GIND.

Using the notation in (D16), the group refractive index μ' is given by the relation

$$\mu.\mu' - \mu^2 = (F/2).d(\mu^2)/dF \qquad \text{(D18)}$$

which leads to

$$(\mu.\mu' - \mu^2)(1-X)D/F = 1 - \mu^2 - X^2 + A.X(1+X)/[2D(1+G^2)^{.5}]. \qquad \text{(D19)}$$

This result is similar to the expression used by Shinn and Whale (1952). Direct evaluation of μ' from (D19) suffers from appreciable small-difference errors under some conditions, primarily in the calculation of $\mu = (1 - FX/D)^{.5}$ and in the evaluation of (1-X) near reflection. The equations were therefore rearranged as follows.

Plasma frequency is provided to GIND in terms of the parameter T of (D17). We then have

$$V = F.(1-X) = F.T^2 \quad \text{for the O-ray,} \qquad \text{(D20a)}$$

and

$$V = F.(1-X) = (F+FH).T^2 - FH \quad \text{for the X-ray (with F negative).} \qquad \text{(D20b)}$$

For the O-ray the value of G is positive, and (D16b) is replaced by the equivalent expression

$$E = A \,/\, [(1 + G^2)^{.5} + G] \qquad \text{(D20c)}$$

The group index is then obtained from

$$W = (F.G.E/V - A/2).(F-V) \,/\, D \,/\, (1+G^2)^{.5} \qquad \text{(D21)}$$

and

$$\mu' = | D + W | \,/\, [\, D.(V + E)\,]^{.5}. \qquad \text{(D22)}$$

The first term in (D21) does not give small-difference errors since it passes smoothly through zero at a point depending on the "constant" A/2. The term F-V tends linearly to zero as the electron density tends to zero (at the base of the ionosphere); at this point W = 0 and (D22) retains full accuracy. Near the reflection point, where T tends to zero, the term F.G.E/V in (D21) becomes much larger than A/2. V also tends to zero so that W and μ' retain full accuracy up to and including the reflection point T = 0. One problem appears at reflection when (D16b) gives G = B/FT^2, for the O-ray. To avoid a division by zero, T^2 is not allowed to fall below 10^{-20}. The exact value of G is not important near reflection, since the terms involving G cancel in (D21) when G is large.

For the X-ray, using a negative value of F, the above equations give the correct value of μ'. A problem arises near reflection, where V is negative and the term V+E in (D22) can give a significant small-difference error. This is avoided if we use, for the X-ray,

$$V + E = -E.U/A/[C + (C^2 + U).5] \qquad (D23)$$

where $U = (V-FH).(F+FH).T^2$ and $C = A + B$.

C is independent of F and of T, so that C and C^2 need be calculated only when the gyrofrequency or the dip angle is altered.

The above equations are implemented in GIND using variables

G1 = V, G2 = G, G3 = $(1 + G^2).5$, G4 = E, G5 = V + E, G6 = D and G7 = W.

Results are normally required for a fixed value of the magnetic dip angle I, while the gyrofrequency FH is changed only occasionally. The speed of the calculations is therefore increased by precalculation of the "constants" A and B in (D16c). This is done in POLAN by a statement

AA = GIND (GH, -DIP)

where GH is the gyrofrequency at ground level. Values of GH, GHSN = GHsinI and GCSCT = $0.5GH(\cos^2 I/\sin I)$ are then calculated and stored in GIND. Note that $\cos^2 I/\sin I$ should not be obtained as (1/sinI - sinI), since the latter form gives unacceptable errors for dip angles near 90°.

Once the ground value of gyrofrequency has been set, the statement

FH = GIND (0., h)

calculates the gyrofrequency FH at the height h, using an inverse-cube extrapolation from the ground value GH. The values of GHSN and GCSCT are also multiplied by FH/GH and stored as FHSN and FCSCT, giving the constants A and B required in (D16). The values of C and of C^2 required for accurate X-ray calculations are also stored at this time. For any required values of wave frequency F, and relative plasma density defined by the parameter T, the group refractive index is then obtained accurately and in minimum time by the statement

$\mu'-1$ = GIND (F,T).

APPENDIX E. THE CONSTRUCTION OF POLAN

The logic used by the main subroutine POLAN, to calculate a real-height profile using ordinary ray data only, is summarised in Section E.1 below. Procedures involved in the use of X-ray data, for start or valley calculations, are given in Section E.2. The interpretation and use of the different parameters is tabulated in Section E.3. Logic flow within the subroutine COEFIC is detailed in Appendix D.1.

INPUT PARAMETERS specified in the call to POLAN are:-

Frequency, Height Arrays FV, HT (and the array dimension N).

Field constants FB, ADIP (in MHz, degrees).

START specifying the starting procedure, as set out in C1.2. Zero gives an extrapolated start, with O-ray data, or a slab start when extraordinary (X-) ray data are present.

AMODE specifies the type of analysis. Zero gives Mode 5 (a fifth order least-squares fit).

VALLEY is normally zero, for a standard calculation of the valley between layers. It can optionally specify a non-standard valley depth, as described in C3.1.

X-rays are identified by negative frequencies and, for each new layer, must precede the corresponding O-ray data.

Within each block, frequencies must increase monotonically - this is used as a check for data errors.

The labels C1, C2, ... below correspond to the numbered comments in the listings of POLAN; labels -(A), -(B), . . refer to sections of a subroutine called by POLAN.

E.1. LOGIC FLOW

E.1.1. PRELIMINARY

C1.1- Store the magnetic field, MODE and VALLEY constants.

C1.2- THE SUBROUTINE SETUP is used to prepare for the START CALCULATION:-

-(A) Check for the presence of initial X-rays, and determine the lowest plasma frequency FMIN in the given data arrays FV, HT.

-(B) Calculate the starting point (FA,HA) for direct, extrapolated, model FS, model HS or X-ray (slab or polynomial) starts. Selection of the start procedure depends on the input parameter "START", as shown below.

(a) With **O-RAY DATA only:**

(i) Value of **START =-1.0** : Use a direct start from the first frequency FMIN, with a real height HMIN equal to the lowest of the first three O-ray virtual heights.

(ii) Value of **START = 0.0** : Use the normal extrapolated start, from a frequency FA which is normally 0.5 MHz, but must be less than 0.6F1.
The starting height HA is obtained by extrapolating the initial virtual-height gradient down to zero frequency; HA must be less than HMIN + 50 km, and greater than HMIN/4 + 55 km.

(iii) **0.0 < START < 45.0** : Use a model plasma frequency FS at a fixed height HS, where:

Range of START =		0-10	10-20	20-30	30-40	40-45	
Start height HS =		90	110	130	150	170	km
Start freq FS =	START minus	0	10	20	30	40	MHz.

(iv) **START ≥ 45.0** : Use a model starting height HA at a fixed frequency FA.
FA is obtained as in (ii) above.
HA is set equal to the value of START, but must not exceed 0.6*HMIN + 0.4*HEXT where HMIN is obtained from (i) and HEXT is the extrapolated height calculated in (ii).

(b) When **X-RAY DATA** are present:

(i) START .GE. -1.0 : Use the standard X-ray slab start.

(ii) START < -1.0 : Calculate an X-ray polynomial start.

-(C) Move the virtual-height data up to start at FV(31), HT(31). Calculated real heights may then begin at FV(1), HT(1), overwriting unwanted (past) virtual-height data as the analysis proceeds.

For extrapolated or model starts, store an additional virtual-height point at FV(30), HT(30). This is at a frequency between the starting point of the polynomial (FA) and the first virtual height (F1), with a height slightly below the minimum observed virtual height. The purpose of this point is to prevent unwanted fluctuations in the real-height polynomial between FA and F1.

Set KV = 30 (or 29) to give the origin FV(KV), HT(KV) of the virtual-height data.

Store the starting point FA, HA at the virtual-height origin (KV) and at the real-height origin FV(KR), HT(KR), where KR = 1.

E.1.2. SELECT DATA POINTS FOR THE NEXT STEP.

C2.1- **Set polynomial constants.**

IF the next step is a new Start, or a Restart (after a cusp or peak),

THEN SET NR = 0 = number of fitted real heights,
NT = number of terms in the polynomial real-height expression,
NV = number of fitted virtual heights,
NH = number of new real heights to calculate,
using elements 1 to 10 of the arrays IT, IV and IH. The element used corresponds to the parameter MODE, reduced to the range 1 to 10.

ELSE SET NR, NT, NV, NH from elements 11 to 20 of the arrays IR,IT,IV,IH.

C2.2- **THE SUBROUTINE SELDAT** is used to

-(A) COUNT the number of initial X-rays (indicated by FV < 0.). If the corresponding plasma frequency at reflection is less than FA, for any X-ray: LIST bad data and EXIT.

CHECK the next NV frequencies. If these do not increase monotonically: LIST bad data and EXIT.

END CHECK IF h' < 0 (implying a cusp)
or IF the next frequency is negative (end of record):
and SET h' = |h'|, FCC = -0.1.

END CHECK IF the next |h'| is less than 30 (implying a peak);
IF the next F = 0, THEN set FCC = +0.1
ELSE set FCC = next frequency.

END CHECK IF the total number of frequencies (X-ray + O-ray) = 30.

-(B) For a START or VALLEY calculation, using X-ray data, add additional points
if (i) the O-ray data does not extend up to the top plasma frequency of the X-ray data,
or (ii) the O-ray frequency range is less than the desired minimum range, as specified by the constant FFIT (in MHz);
PROVIDED THAT the added point does not have excessive group retardation; the virtual-height gradient $\Delta h'/\Delta F$ must be less than the constant GFIT, in km/MHz.

-(C) Shift the data arrays to delete any X-ray data for which FN (at reflection) is greater than FM + 0.1 MHz.
Set FM = FV(MF) = the highest O-ray frequency to be used in this step.

IF KR = 1 (indicating the first step in the analysis): GO TO C3.

C2.3- **THE SUBROUTINE REDUCE** is used to reduce all virtual heights by the group delay in the last-calculated real-height section.

-(A) Reduction using the full polynomial expression for the real-height profile.

Get the index KM for the highest frequency with FN (at reflection) less than FM + 0.04*MODE, and with h'> 30 km (i.e. not a cusp or a peak).
[This gives FN up to FM + 0.2 MHz for the normal default Mode = 5,
or FN up to FM + 0.6 MHz for increased accuracy at Mode = 15].
CALL COEFIC with the first parameter negative, to calculate the total group delay (integrating from the previous origin FA to the current origin FV(KV)), and subtract this from the virtual height for each frequency from FV(KV+1) to FV(KM).

-(B) Reduction using separate laminations.

For each of the newly calculated real-height intervals, and for frequencies F from FV(KM+1) to the end of the data, determine the effective mean plasma frequency FAV. The optimum value of FAV depends on the difference F-FAV; on the change in gradient across the real height interval; and on the possible presence of a peak (infinite gradient) at the upper limit.
Calculate the mean group refractive index, corresponding to (F, FAV), and subtract the corresponding group retardation from the virtual height h'(F).

IF $|h'| < 30$ (denoting a critical frequency) omit the calculation, leaving h' unchanged.

IF $h' < 0$ (denoting a cusp) add the calculated group retardation to h'.

E.1.3. SET UP SIMULTANEOUS EQUATIONS FOR THE NEXT PROFILE STEP

C3.1- **INITIALISE**

SET the number of polynomial terms MT = NT + (NX+1)/2, where
NX is the number of X-rays included in this step,
MT must not exceed 15,
MT must not exceed the total number of fitted points (virtual + real).

SET the origin at FA = FV(KR), HA = HT(KR).
SET JM = MT = Total number of terms in the real-height expression.

IF an X-ray start, OR the calculation is restarting above a peak (with a possible valley),
THEN SET JM = MT+1 to include a constant (offset) term in the real-height polynomial.

VALLEY check (within the subroutine STAVAL). The end of the data for any one layer is indicated by a critical frequency (scaled or zero) accompanied by a virtual height which is less than 30 km in absolute value.
IF the virtual height HV(KV) (at the origin frequency FA) is greater then 30 km,
THEN SET HVAL = 0. and GO TO C3.2 (for a normal step)
ELSE carry out the valley set-up procedure described in Appendix E.2 (1) below.

C3.2- **CALCULATE COEFFICIENTS** for the simultaneous equations defining the real-height profile.

CALL COEFIC (MV, FV, HT) to calculate the coefficients B(i,j), A(i,j) such that

$$\sum_{j=1}^{JM} B(i,j).Q_j = HT(KV+i) - HA \quad \text{for fitted virtual heights,} \quad i = 1 \text{ to } MV.$$

$$\sum_{j=1}^{JM} A(i,j).Q_j = HT(KR+i) - HA \quad \text{for fitted real heights,} \quad i = 1 \text{ to } NR.$$

The real-height profile from FA = FV(KV) [= FV(KR)], to the highest reflected frequency FM = FV(KV+NV), is assumed to be of the form

$$H - HA = \sum_{j=1}^{JM} Q_j.(F - FA)^j, \quad \text{so that} \quad A(i,j) = (FV(KV+i) - FA)^j.$$

IF JM = MT (the number of polynomial terms in the real-height expression):
THEN GO TO C4.
ELSE add a constant term, to allow for a shift in the real-height origin in start and valley calculations. This logic is described in Appendix E.2 (2) below.

E.1.4. LEAST-SQUARES SOLUTION of the simultaneous equations, in the Matrix **B**.

C4.1- **CALL SOLVE (NS, JM, B, Q, DEVN)** to obtain the least-squares solution of the NS = MV+NR+MS equations. This solution gives the coefficients Q(1) to Q(JM) defining the real-height profile, and the R.M.S. deviation DEVN of the least-squares solution (when JM < NS).

The real-height equations are given a large weight in the least-squares solution, so that DEVN gives closely the R.M.S. fitting accuracy (in km) for the virtual-height data.

C4.2- **CALL ADJUST** to check the solution as follows. (Note that these adjustments can be omitted by using a negative value for the input parameter DIP).

-(A) The **INITIAL GRADIENT Q(1)** of the polynomial real-height section.
For normal steps the minimum physically-reasonable gradient is MINQ1 = 1.5 km/MHz.
For valley steps (HVAL non-zero) the minimum initial gradient of the real-height polynomial, above the valley, is MINQ1 = SHA (= the model atmospheric scale height).
IF Q(1) < MINQ1
THEN add the constraint Q(1) = MINQ1 and obtain a new least-squares solution.

For start calculations using X-rays, an upper limit is placed on Q(1), to reduce meaningless variations in the unobserved region at FN < F1:-
IF Q(1) > 100 km/MHz, and LK < 1
THEN add the constraint Q(1) = 100, and obtain a new least-squares solution.

-(B) If the **number of terms (MQ)** used in the real-height polynomial is larger than necessary, the high order terms are not well defined by the least-squares solution. Checks are carried out for this condition, and the order of the polynomial is reduced (unless this is the last real-height section before a peak, when a rapid change in gradient is normal and the checks are omitted).

IF the last three (high-order) polynomial coefficients alternate in sign, with an overall increase in magnitude of more than a factor of 2,
or IF the magnitude of either of the last two coefficients exceeds 999.,
THEN add the constraint Q(MQ) = 0, and obtain a new least-squares solution.

This check is repeated until (i) the number of non-zero polynomial terms is less than 5, or (ii) the magnitude of the last polynomial term is less than 150.

-(C) Corrections in the **START or VALLEY** regions.

(i) Limit the thickness of the underlying slab used in an X-ray start calculation.
IF the thickness Q(MT) of the calculated linear slab of low-density ionisation is negative,
THEN impose the constraint Q(MT) = 0 and obtain a new least-squares solution for the other real-height parameters Q(j).
IF the new solution increases the R.M.S. virtual-height fitting error by more than 25%,
THEN return to the original solution with Q(MT) negative.

(ii) START calculations: the real height at the start of the polynomial section (at the frequency FA) must be less than the original height HA, and greater than 60km.
IF the calculated shift Q(JM) in the real height at the origin (FA,HA) is positive,
THEN obtain a new solution with Q(JM) = 0.
IF the calculated shift Q(JM) lowers HA to less than 60 km,
THEN impose the constraint Q(JM) = 60 - HA, and recalculate the least-squares solution.

(iii) VALLEY calculations (using O- or X-rays):
IF the calculated shift Q(JM) in the height at the frequency FA is negative,
THEN obtain a new least-squares solution with Q(JM) = 0.1

C4.3- **ITERATION** of valley calculations, to adjust VDEPTH;
or of X-start calculations, to adjust the gyrofrequency.

C4.3- **ITERATION** of valley calculations, to adjust VDEPTH;
or of X-start calculations, to adjust the gyrofrequency.

IF JM = MT+1, AND the number of X-rays is zero (implying an **O-RAY VALLEY CALCULATION**)
THEN the total valley width is VWIDTH = PARHT + (calculated value of Q(JM));
IF this is the first solution for this valley,
THEN GO TO X1.C (in STAVAL) to adjust VDEPTH to correspond to the standard valley shape, and recalculate the solution.

IF X-rays are used, in the **START or VALLEY calculation,** additional checks are carried out within STAVAL, as outlined in section E.2.3 below. Results for ionisation in the unobserved start or valley regions are then listed and, if necessary, iterated with altered values for VDEPTH or for the gyrofrequency height FHHT (as in section E.2.3).

E.1.5. CALCULATE AND STORE REAL HEIGHTS at the next NH frequencies, setting

FV(KT) = FV(KA), where KT = KR+NR+1 to KR+NR+NH; KA = KT + (KV-KR); and

$$HT(KT) = HA \sum_{j=1}^{MQ} Q_j.(FV(KT) - FA)^j.$$

MQ is equal to MT for all conditions except for a slab start calculation, when MQ = MT-1 [and the term Q(MT) gives the thickness of the underlying slab].

IF for any calculated point the height HT(KT) is less than HT(KT-1), and Q(MQ) has not already been altered,
THEN PRINT "Error at freq = FV(KT)";
Obtain a new least-square solution with the additional constraint Q(MQ) = 0.0 (effectively reducing the order of the fitted real-height polynomial by one);
Recalculate HT(KT), and continue with the analysis.

CALCULATE further real heights, up to the highest virtual-height frequency used in the analysis, from the polynomial expansion. (These heights will be recalculated more accurately in a later step. Preliminary values may, however, be used by the later steps to obtain the correct gyrofrequency.)

IF FC is zero (i.e. the calculation did not extend to a peak or cusp)
GO TO C2.1A to calculate the next real-height step.

IF FC is positive, GO TO C6 to calculate the parameters of the peak.

IF FC = -0.1, GO TO C2.1 to restart calculations above the cusp.

IF FC is negative, GO TO C7.3 to end the analysis.

E.1.6. LEAST-SQUARES CALCULATION OF A CHAPMAN-LAYER PEAK.

The critical frequency and the scale height of the peak are obtained by fitting a Chapman layer to the calculated real-height gradients. This fit normally requires iteration, since height h cannot be expressed directly in terms of the plasma frequency FN. The calculations begin with an assumed model value SHA for the scale height. After two calculations we get a result which is almost independent of SHA, if virtual heights have been scaled close to FC. As the interval between the highest scaled frequencies and the critical frequency increases, giving less information on the scale height near the peak, the calculated scale height tends more towards the model value. With poor data the calculation is not iterated, and results remain more heavily weighted towards the model value SHA.

C6 --
-(A) SET NK equal to the number of data points (MV) used in the last real-height fit, giving the number of data points to use in the peak fit calculation.

IF MODE < 4 (so that MV < 3) use NK = MV + 1.

IF foFC was scaled, increase NK by one to include this measurement.
IF fxFC was scaled, increase NK by one to include this measurement.

SET **INITIAL VALUES:** **Scale height** SHA = HM/4 - 20 km, and **Peak height** HM = HN + 0.3*SHA, where HN = h(FM) is the last calculated real height.

SET FW = F1 - ΔF/2, where ΔF is the frequency range used in the peak fit.

-(B) For each frequency F_i included in the peak fit:

CALCULATE the normalised gradient Grad = (4/FN).d(FN)/dh, from the last real-height polynomial.

CALCULATE coefficients **B(i,2)** for the equation

$$\mathbf{B(i,1)}.\ln(FC) + \mathbf{B(i,2)}.(SH/SHA)^2 = \mathbf{B(i,3)}.\ln(F_i),$$

using the Chapman-layer assumption **B(i,1)** = **B(i,3)** = 1.

Multiply the **B(i,j)** by $W_i = (F_i - FW)/(FM - FW)$, where FM is the highest scaled frequency; this gives a weight proportional to $(F_i - FW)^2$ in the least-squares solution.

-- For each scaled critical frequency, add a peak fit equation with **B(i,2)** = 0.

IF the given FC is an X-ray measurement, convert it to the corresponding plasma frequency (using the gyrofrequency at the height HM).

-- SOLVE the NP simultaneous equations, to give the values of **ln(FC)** and **SH.**

IF the real-height **gradient dh/dFN** increases by less than 40% over the top half of the fitted frequency range (so that the profile curvature and hence the scale height are not adequately defined),

THEN replace SH by (SH+SHA)/2 when SH < SHA, or by 2.SH.SHA/(SH+SHA) at SH > SHA, and modify the least-squares fit to obtain the corresponding value of FC. (This adjustment is cancelled if it increases the R.M.S. deviation of the calculation by a factor of more than 2.0.)

-- USE SH and the gradient at FM to calculate the peak height HM.

IF the extrapolation range HM - h(FM) is greater than 1.8 scale heights,
THEN set HM = h(FM) + 1.8 SH.

IF the peak extrapolation exceeds one scale height, or the real-height gradient increases by less than 80% over the top half of the fitted frequency range,

THEN GO TO -(C) (leaving the calculated SH weighted towards SHA).

ELSE IF the peak fit has been performed once only, set SHA = SH and GO TO -(B) (to recalculate SH, FC using the updated values of SH, HM).

-(C) Final **adjustment and listing of the peak constants.**

Calculate the critical frequency FPAR for a parabolic peak with height HM and scale height 1.25SH; The frequency extrapolation FC-FM must agree to within a factor of 2 with the range FPAR-FM.

The least-squares fit gives a R.M.S. deviation for the calculated value of ln(FC); convert this to a standard error in FC, and print the peak constants.

E.1.7. CONTINUE with a new layer; or TERMINATE.

IF the next virtual height is not zero,
THEN GO TO C2 (do next profile step).

ELSE set N equal to the number of real-height data points in the arrays HT, FV. Store the scale height of the final peak, the standard error in FC, the overall slab thickness, the total electron content, the width of the last valley and the R.M.S. deviation of the last X-ray fit in the following three elements of the arrays HT, FV. (The total content and slab thickness are obtained by exact numerical integration of the analytic real-height expressions.)

RETURN FROM POLAN.

E.2. START AND VALLEY PROCEDURES

When X-ray data are used for start or valley calculations, additional logic is required to adjust the solution and to iterate it as required. Most of the logic relating to start and valley calculations is contained in the subroutine STAVAL. This is first called from section C3.1 of POLAN, to set up the valley constants as described in E.2.1 below. After formulation of the basic real height equations, additional equations are added to apply physical constraints to a start or valley solution. This is done in Section C3.3 of POLAN and is outlined in E.2.2 below. After the real-height solution has been obtained STAVAL is called again, from Section C4.2 of POLAN, to check the result and repeat the calculation if required. The logic for this process is summarised in Section E.2.3.

E.2.1. INITIAL VALLEY CONSTANTS

In the absence of X-ray data the type of valley calculation is normally defined by the parameter VALLEY in the call to POLAN. Internally this is used to set the valley flag parameter HVAL. If a non-zero virtual height is given at the critical-frequency data point, this height is used (instead of VALLEY) to define the value of HVAL for the current profile. The following steps describe the logic used to set initial constants for the valley region, in section C3.1 of POLAN.

(a) **The valley flag HVAL:-**

SET HVAL = HV(KV) or, if this is zero, SET HVAL = input parameter VALLEY.
IF HVAL = 0.0 (the default condition), THEN SET HVAL = 1.0 for a normal valley.
IF HVAL .GE. 10.0, SET HVAL = 0.0 so that a valley is not inserted above the peak.

(b) **The valley width VWIDTH:-**

IF HVAL = 1.0, SET VWIDTH equal to the standard value of 2*SHA km,
where SHA = (peak height)/4 - 20 = model atmospheric scale height.
IF HVAL is in the range 0.1 to 5.0, the value of VWIDTH is multiplied by HVAL;
this is used to scale the standard valley.
IF HVAL < -1.0, SET VWIDTH = 5 * INT|HVAL|;
this sets a required value of valley width, defined in 5 km steps.
Any decimal part of HVAL defines the valley depth as in (c) below.

(c) **The valley depth VDEPTH:-**

SET VDEPTH equal to the standard value $0.155*VWIDTH^2/(VWIDTH+20)$ MHz.
IF HVAL < 0.0 and HVAL is not integral, SET VDEPTH equal to the decimal part of |HVAL|.

IF NX > 0 (an X-ray calculation),
AND HVAL = -1.0 (requesting a determination of valley width and depth),
THEN SET VDEPTH = 0.1001 MHz, as an initial depth from which to iterate.

SET VDEPTH = VDEPTH*FA/(VDEPTH+FA), to ensure that the valley depth is less than the critical frequency (FC, = FA) of the underlying peak.

(d) **The parabolic PEAK section:-**

Constants for the underlying peak are the critical frequency FC = FA, the peak height HMAX and the scale height SH; these have been calculated in the previous real-height step of POLAN.

EXTRAPOLATE the underlying peak to FN = FC - VDEPTH, using a parabolic section with a scale height of 1.4*SH.

SET the origin at HA = HM + PARHT, where PARHT is the thickness of the extrapolated peak section.

SET the mean gyrofrequency height FHHT = HA + 20 km (for variable gyrofrequency calculations).

E.2.2. THE ADDITION OF PHYSICAL CONSTRAINTS

For **START** and **VALLEY** calculations the total number of terms in the real-height polynomial is JM = MT + 1. The extra term allows a calculated shift in the height at the frequency FA. To aid in reliable calculations of this shift, a number of "physical constraints" are added to the set of simultaneous equations. These are applied in Section C3.3 of POLAN, and serve to bias the results to a more physically acceptable form. (Note that these physical constraints can be omitted by making the input mode parameter AMODE negative).

(a) IF **LK < 0** we have an **EXTRAORDINARY-RAY START CALCULATION.**

The assumed real-height expression is then: $$H - HA = \sum_{j=1}^{MT-1} Q_j.(F - FA)^j + Q(JM)$$

The additional (constant) term Q(JM) gives the amount by which the calculated real height differs from the initial estimate HA at the starting frequency FA. The corresponding virtual height term is B(i,JM) = 1.

The term involving Q(MT) does not appear in the real-height polynomial. This term gives the thickness of an underlying, low-density slab of ionisation with FN increasing linearly from 0.3F1 to 0.6F1 (= FA), where F1 is the lowest observed plasma frequency. The virtual height term B(i,MT) gives the group retardation produced by this slab.

Three physical constraints are currently added to the set of simultaneous equations for an X-ray start calculation. These are given a small effective weight so that they alter the result only if the virtual-height equations do not, in themselves, give a well-defined solution. The added equations require that:

(i) The calculated offset Q(JM) gives approximate agreement with the starting height which would be used in the absence of X-rays (the height HS obtained by SETUP in section C1.2).

(ii) The slab thickness Q(MT) is approximately equal to HS/3-20 km.

(iii) The high order term Q(MT-1) in the polynomial real-height expression should be small.

(b) IF **HVAL is not zero,** we have a **VALLEY CALCULATION,** using O-rays or combined O- and X-rays.

The real-height expression is then: $$H - HA = \sum_{j=1}^{MT} Q_j.(F - FA)^j + Q(JM)$$

where Q(JM) gives the amount by which H(FA) exceeds HA. Thus the calculated value of Q(JM) gives the total valley width less the distance PARHT (= HA-HM) corresponding to the extrapolated parabolic peak in C3.1(D).

The virtual-height relations are: $$\sum_{j=1}^{JM} B(i,j).Q_j = HT(KV+i) - HA - P_i$$

where P_i is the virtual-height increase caused by the extrapolated parabolic peak section at the frequency F_i = FV(KV+i). B(i,MT) gives the virtual-height increase from the upper part of the valley. This consists of:-

(i) A region of constant plasma frequency FN = FA - VDEPTH, with a thickness 0.6*Q(JM), and

(ii) A region with FN increasing linearly from FA - VDEPTH to FA, over a distance 0.4*Q(JM).

Valley calculations include four physical constraints to bias ill-defined results towards a physically reasonable model. For ordinary ray calculations, virtual-height data give no information about the valley size. The added physical relations are then given the same weight in the least-squares solution as the virtual-height data. With combined O- and X-ray data, which can (ideally) define the valley width without ambiguity, the relation (i) is given a smaller weight. The added equations (described more fully in Section 7.3 of this report) specify that

(i) Q(JM) = VWIDTH - PARHT. The calculated valley width should be equal to the "standard" value from C3.1(b). This constraint has a weight of 1.0 under normal conditions. The weight is increased to 100 when HVAL < -1.0, indicating that a specific valley width is required. When X-ray data are used to determine the valley width, the weight is reduced to 0.04.

(ii) The gradient at the top of the valley section should match the gradient at the start of the polynomial section. This condition has a weight of 0.16.

(iii) The high order polynomial term Q(MT) should be small. This condition has a weight of 0.25.

(iv) The term Q(MT-1) should be small. This condition has a weight of only 0.02.

E.2.3. ITERATION OF THE PROFILE

In start or valley calculations using X-ray data, the calculated profile must be iterated to allow for height variations in the value of the gyrofrequency FH, and to adjust for changes in the valley depth. These adjustments are controlled by the subroutine STAVAL, after calculation of the parameters Q(1) to Q(JM) which define the real-height profile and after the elimination of unphysical values of these parameters, but before the real heights are stored.

Labels CX... below identify the corresponding section in the subroutine STAVAL.

CX2- **CALCULATE THE REAL HEIGHTS** at all frequencies used in the analysis, from the coefficients Q.

IF START = -0.1, **THEN** GO TO CX3 (omitting checks).

IF the number of iteration cycles NC is greater than (about) 10 for this step,
THEN GO TO CX3 (continue with no further adjustments).

IF the profile gradient $(h_i - h_{i-1})/(F_i - F_{i-1})$ is greater than 2 at all points,
THEN GO TO CX3 (continue with satisfactory profile).

TREATMENT OF FAULTY PROFILES:-

IF the X-ray data had a weight XWAT = 1, and NC > 1;
THEN SET XWAT = 0.5; PRINT "X-ray weights reduced to 1/4"; and
GO TO C3.2 to recalculate the profile.

IF this frequency interval (F_{i-1} to F_i) has already caused downwards adjustment of Q(JM),
AND the adjustment failed to increase $h_i - h_{i-1}$;
THEN PRINT "data and gyrofrequency incompatible at $F = F_i$" and continue with no further adjustments or iteration.
ELSE modify the least-squares solution by imposing a smaller value of Q(JM) (lowering the start point); and
GO TO CX2.

CX3- **SET FHHT** = the height of an X-ray about one third of the way up the fitted range.
IF the calculation involves X-rays, with a height-variable gyrofrequency,
AND the gyrofrequency height has changed by more than 2.0 km,
THEN SET LOOP = 3 so that the solution will be iterated from C3.2.

CX4,5-
LIST the Start, Valley constants.

IF LOOP = 3, GO TO C3.2 (recalculate using FB at the last-found heights).

IF this is a start calculation, GO TO CX7.

CX6- X-RAY VALLEY ADJUSTMENTS.

CX6.A **SINGLE-PARAMETER CALCULATIONS** determine the width of a standard shaped valley.
IF the solution has not been iterated,
THEN Adjust the depth to correspond to the calculated width, in C3.1(c).
Recalculate the thickness of the peak section, in C3.1(d).
GO TO C3.2 to recalculate the profile and the valley width.
ELSE IF the depth adjustment increased the fitting error DEVN by more than 10%,
THEN recalculate the profile and valley width using the original depth.
ELSE GO TO CX7 to store the valley heights.

CX6.B **TWO-PARAMETER CALCULATIONS determine Valley Width and Depth** independently (when VALLEY < 0).
The first two iterations use depths VAL of 0.1001 and 0.6006 MHz. Further steps continue from whichever solution gives the smaller R.M.S. deviation (DEVN), adjusting VDEPTH by a factor DVAL = 1 + 0.5 cos(0.85DIP) at each step. After each adjustment of VAL, the entire real-height calculation must be repeated.

IF VAL = 0.1001: set VAL = 0.6006 MHz; set DEVL = DEVN; and **RECALCULATE VALLEY PROFILE**
(go to C3.1(d), in STAVAL, to adjust the peak thickness and recycle).
IF VAL = 0.6006:
THEN IF DEVN < DEVL, THEN set DEVL = DEVN.
ELSE set VAL = 0.1001 to revert to the smaller depth.
SET VAL = VAL*DVAL and RECALCULATE VALLEY PROFILE.

IF DVAL is negative, GO to CX7 (end of valley iteration).

IF this is the THIRD CYCLE:-
THEN IF DEVN > DEVL
THEN set DVAL = 1./DVAL (to reverse the direction of iteration)
set VAL = VAL*DVAL^2.
ELSE set VAL = VAL*DVAL, DEVL = DEVN.
RECALCULATE VALLEY PROFILE.

IF DEVN < 0.97 DEVL - 0.003 KM, on LATER CYCLES,
THEN set VAL = VAL*DVAL, DEVL = DEVN.
ELSE we have passed a minimum (or DEVN is changing very slowly);
INTERPOLATE in the last three values of DEVN to find the minimum, limiting the result to within a factor DVAL$^{1/2}$ of the last interval.
Calculate the corresponding value of VAL, and set DVAL = -DVAL (to signal the final calculation).

RECALCULATE VALLEY PROFILE by doing C3.1(d), within STAVAL, followed by GO TO C3.2 in POLAN.

CX7- STORE DATA POINTS for the start or valley region, at frequencies less than FA, and at F = FA.

INCREASE KV to the first O-ray point (so that real heights will not be calculated at the X-ray frequencies).

GO TO C5 to calculate, check and store the real heights at F > FA.

END X RAYS

E.3. PROGRAM PARAMETERS

The relation between many parameters varies depending on the stage reached in the real-height analysis:- a normal step, a start calculation using O-ray data only, start using O- and X-ray data, valley using O-ray data only, or valley using O- and X-ray data.

Table E1 summarises the values and interpretation of some of these parameters under different conditions. The labels C1, C2, ... in this table refer to the corresponding section in POLAN. KR and KV are the indices for the current origin, giving the position of the point FA, HA in the real and virtual-height data arrays FV, HT.

Table E2 shows how different variables and parameters are used. The first column in this Table identifies the section within POLAN. The second column shows variables which are tested to determine a course of action. Program changes which alter any of the variables in this column will alter the path of the program. The last column in Table E2 shows how variables are changed at different stages of the program. This provides a ready reference to the setting, use and meaning of the variables within POLAN.

TABLE E1. Interpretation of parameters, according to the type of the current real-height step.

	0 START	X - START Slab Poly	VALLEY 0 X	PROFILE STEP Normal Cusp; Peak
C1.				
LK	1	-1 0	>2	>2
JS	2 (0=direct)	1	-	-
NX	0	>0	0 0	0
KR	1	1	>1	>1
C2.	NF = No of 0 rays; MV = Total Frequencies (=NF+NX); FM = FV(MF) = Top Freq.			
MOD	<11	<11	<11	>10
HT(KV)	>30	>30	<30	>30 <-30;|h'|<30
FCC	0	0	0	0 -.1; .1 or FC
NX	0	>0	0 >0	0
reduce?	no	no	yes	yes
C3.	MT = Number of terms in the real-height expansion (omitting any constant) Parameters after JM are set in STAVAL.			
JM	MT	MT+1	MT+1	MT
KD	0	(0)	3	0
HS	-	-	HM+SHA	-
HVAL	0	0	>0	0 0
VDEPTH	0	.3F1 0	>0	0
PARHT	0	0	>0	0
C3.2	Call **COEFIC** (with MV frequencies) to calculate B(I,J).			
C3.3	Add "**Physical Relations**" equations.			
C4.	Call **SOLVE** to obtain the real-height parameters Q(1) to Q(JM).			
C4.2	Call **ADJUST** to limit the parameters Q to physically acceptable ranges.			
C4.3	For **START** or **VALLEY** Calculations: Do Section CX. below (in STAVAL).			
C5.	Check and **Store Real Heights** HT(KT), for KT = KR+NR+1 to KR+NR+NF, at frequencies FV(KV =NR =1) to FV(MF) = FM.			
KT	KR+NF	KR+NF	KR+NF	KR+NF
KR	KR+NH	KR+NH	KR+NH	KR+NH KT ; KT+1
KV	KV+NH	KV+NH	KV+NH	KV+NH KV+NH;KV+NH+1
C6.	**Layer Peak**: If FC > 0., Calculate and store HT(KR) = HMAX GO TO C2.			

CX. -- in STAVAL	**X RAY CALCULATIONS**	
	START	**VALLEY**
Q(MT) (= Slab)	≥0 -	- -
Q(JM) (= Offset)	≤0 ≤0	≥0 ≥0
Q(JM) if dh/dFN >2	decrease Q(JM)	decrease Q(JM)
	Check Heights	Check Heights
	Iterate FB	Iterate FB, VDEPTH
KD	2 1	3 3
KR	2 1	KR+3 KR+3
KV	KV+NX-1	- KV+NX
NH	NH+1 NH+2	- -

TABLE E2. Testing and setting of parameters. Quotes denote an "Input Parameter" to POLAN.

S E C T I O N	C O N D I T I O N - Variables tested.	A C T I O N - Variables Altered.
C1. PRELIMINARY	"AMODE"	MODE (1 to 20), MOD1 (1 to 10).
C1.2 (in **SETUP**)	"FV(1,2,3, ...)"	FMIN = lowest plasma frequency in data.
Normal Start	"START" >= 0	LK = 1, JS = 2, HS = extrapolated.
Direct Start	"START" = -1.0	HS = h'min, FA=F(1), JS=0, LK=1.
Model Height	|"START"| >= 44	HS = model.
	All steps	HA = HS, FA = 0.5.
Model Density	|"START"| < 44 (and not zero)	HA, FA are given the model values.
X Start: Slab	FV(1) < 0.	LK = -1, JS = 1, HA, FA, VDEPTH.
POLY	FV(1) < 0. and "START" < -44.	LK = VDEPTH = 0.
	All steps	FV(1, 29-), HT(1, 29-).
	All steps	KR = 1, KV = 29 (for a normal start)
		30 (X ray start)
		31 (direct start)
C2. SELECT DATA		
C2.1	Start or cusp/peak restart	MOD = 1 to 10; NNR = 0; KT = KR
C2.1A	Second step	MOD =11 TO 20; KR, KV are reduced.
C2.1B	All steps	set NT, NV, NH, NR, NL
		set FHHT and FH; set FCC = 0 (no peak).
C2.2 (in **SELDAT**)		NX = number of X rays
		MV = total number of frequencies fitted
		FM = FV(MF) = FV(KV+MV) = top frequency
	HT(MF) < 0 (a cusp)	FCC = -0.1, HT(MF) = |HT(MF)|
	FV(MF+1) < 0 (data ends)	FCC = -0.1
	HT(MF+1) ≤30 (a peak)	FCC = AMAX(0.1, scaled FC).
	Top X-rays > FM + 0.1 MHz	Reduce NX and MV to delete rays,
		move FV(K), HT(K) at K >= KV+NX.
	NX = 0	LK >= 1
C2.3 (in **REDUCE**)		
C2.A (COEFIC)	At KR > 1	KM = top point for exact reduction;
		HT(KV+1 to KM) are reduced
	LK = -1 or 0	LK = 1
C2.B	At KR > 1	HT(KM+1 to end) are reduced
		LK = KR
		FA = FV(KR), HA = HT(KR)
C3. Set Constants		
C3.1	All steps	JM = MT = number of terms in polynomial
	All steps	FC = FCC, KD = 0
	All steps	NC = MC = 0 (Iteration counters)
	|HT(KV)| > 10.0	HVAL = 0.; Go to C3.2 (no valley)
Valley Constants		
CX1. (in STAVAL)		DEVLL = 10^7 = 10*DEVL.
		SHA = HM/4-20 (model scale height).
		HS = HM + SHA (field height).
CX1.B		VWIDTH = 2.SHA.
	HVAL < 9	VWIDTH = VWIDTH*HVAL.
	HVAL < -1.	VWIDTH = 5.*INT(-HVAL)
		VAL = Standard Depth
	HVAL < 0. and NX > 0	VAL = 0.1001 MHz (initial trial).
		VDEPTH = VAL limited to < FA
		PARHT = ht above peak, to FC-VDEPTH
		HA = HM+PARHT (start of slab valley)
	NC = 0 (first cycle)	FHHT = HA+20 km (valley field height)
	LK ≤ 0 (X start) or HVAL ≠ 0	JM = MT+1 (constant term; JM ≤ MV)
C3.2 **Form equations**		Coefficients B(I, 1 to JM+1) are
		stored by COEFIC, for I = 1 to MV+NR.
		Add physical equations to B(I,J).
		NC = count of cycles through C4.
C4. Solve equations		Real-height coefficients Q(1 to MT).
C4.2 (in ADJUST)	LK = 0 or 1, and Q(1) > 50	Re-solve with Q(1) = 50 km.
	Q(1) < 2.0	Re-solve with Q(1) = 2.0 km.
C4.3 (in STAVAL)	JM > MT (X start, or any valley)	Go to CX1 (X-ray checks and iteration)