ON REDUCING THE TIME IN MAGNETIC RESONANCE PROCESSES

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ABSTRACT

This paper gives some results on the studying to solve the Bloch equations in rotating reference system by applying the FDTD method. The FDTD structure converted into an appropriate inverse problem in time domain. The solutions and variations of several MRI processes with respect to the specific parameters are displayed. These specific parameters are related to diagnose an illness. Various computational results related with several healthy and/or ill organs and/or conditions are given. This study provides the possibility of scanning smaller region then the region required in common MRI applications in use; beside, provides the possibility of spending lesser time then the time spent already.

INTRODUCTION

Both of MRI and NMRI have a place in several areas of applications; i.e., medicine, remote sensing, material recognition, various detection processes in both industry and military defense systems, and, etc. Time domain versions of MRI processes have a potential at adding new viewpoints to these processes besides giving the improving tools to produce MRI devices. As it is known, MRI and NMRI are related with the analysis of electrical signals induced on the specific sensors due to the magnetic spin of nucleus and/or molecules and magnetic momentum of the electron in its orbit, respectively. Some of MRI and/or NMRI systems use these magnetic fields rather then the electrical signals by transferring them to the acoustic pressure. All the processes involving magnetic resonance are given with Bloch equations and uses rotating reference system [1]-[3].

This paper gives some results on the studying to solve the Bloch equations in rotating reference system by applying the FDTD method [4]-[6]. Displaying the solutions and variations of several MRI processes with respect to the specific parameters are within the interest of this study. These specific parameters are related to diagnose an illness. This study is aiming to become shorter the time spent during the scanning. The 3D imaging process is generated as a side consequence of the approach used here.

The study involves two parts. The first part is about extracting the MRI figures in continuous forms in time domain during the predefined time range. Beside, getting the variation of these figures with respect to time is in the area of interest. The second part is about extracting the results expected from MR applications but increasing the performance of MR devices.

Bloch equations are written by using FDTD approach. The FDTD structure converted into an appropriate inverse problem in time domain. The configuration of this inverse problem is arranged in a suitable form, which will generate the magnetization vector, by the use of measured magnetic resonance signal. These equations are nonlinear; however, the input vector is transferred into a suitable form such that obtaining a stable, convergent and one-to-one solution is provided. Various computational results related with several healthy and/or ill organs and/or conditions are given. This study provides the possibility of scanning smaller region then the region required in common MRI applications in use; beside, provides the possibility of spending lesser time then the time spent already. Both of these opportunities will contribute a lot of help on the patients. An example of the results is given below:

If we accept a specific resolution and get one measure at a second then reducing the scanning time to a few minutes is possible to collect the data for regenerating the MR images.

THE FORMULATION OF THE PROBLEM

The full Bloch equations in rotating reference system are below:

$$\frac{\partial}{\partial t}\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \begin{bmatrix} -\omega_0 \frac{\sin(2\omega_1 t)}{\cos(2\omega_1 t)} + \frac{1}{T_2} & -\omega_0 & \gamma(\omega_0 - \omega_1)B_1 \frac{\sin(\omega_1 t)}{\cos(2\omega_1 t)} \\ \omega_0 / \cos(2\omega_1 t) & 1/T_2 & -\gamma(\omega_0 - \omega_1)B_1 \frac{\cos(\omega_1 t)}{\cos(2\omega_1 t)} \\ \gamma B_1 \sin(\omega_1 t) & \gamma B_1 \cos(\omega_1 t) & -1/T_1 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/T_1 \end{bmatrix} M_0 \qquad (1)$$

Here $[M_x \quad M_y \quad M_z]^T$ is the magnetization vector and $\omega_0 (= \gamma B_0)$, ω_1, γ, B_1, T_1, T_2, and M_0 are Larmor frequency, angular frequency of the RF field, gyromagnetic ratio, amplitude of external magnetic induction vector varying on the plane z=0 with angular frequency ω_1, spin-cage relaxation, spin-spin relaxation, and balance magnetization, respectively. The B_0 is the amplitude of static magnetic induction vector and superscript T illustrates the matrix transposition.

Equation (1) provides the below finite difference equations in time domain, where t_0, n, and Δt illustrate the initial time, time step, and time difference:

$$\begin{bmatrix} M_x^{n+1} \\ M_y^{n+1} \\ M_z^{n+1} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\Delta t}{T_2^n} & \gamma^n \Delta t(B_0 - B_1^n) & 0 \\ -\Delta t(B_0 - B_1^n) & 1 - \frac{\Delta t}{T_2^n} & \gamma^n \Delta t B_1^n \\ -\gamma^n \Delta t\, B_1^n \cos[\omega_1 (t_0 + n\Delta t)] & -\gamma^n \Delta t\, B_1^n \cos[\omega_1 (t_0 + n\Delta t)] & 1 - \frac{\Delta t}{T_1^n} \end{bmatrix} \begin{bmatrix} M_x^n \\ M_y^n \\ M_z^n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\Delta t}{T_1^n} \end{bmatrix} M_0 \qquad (2)$$

INVERSION APPROACH USING FDTD SCHEME

Let us focuse on diagnosis at a specific location, where we get a suspecion about a specific desease. Is it possible to diagnose by treating this specific location, where only a few points are scanned? If we select one quartet (x_1, y_1, z_1, t_1) of spatial and temporal coordinates on the organ under treatment then the answer is positive by taking a few different initial times and temporal steps and spatial steps on one coordinate direction. The amplitude B_1 of external RF field should be changed to B_{1i} for every triplet of $(t_{i0}, \Delta t_i, \Delta x_i)$ for i=1, 2, 3, …, q, where the number of this triplet is q.

The equation below is obtained by considering the second order terms in Δt, where q=11:

$$[Q]_{11\times 11}[M]_{11\times 1} = [S]_{11\times 1} \qquad (3)$$

Here the $[M]_{11\times 1}$ is unknown column matrix and has the rows below, where I, J, and K are the spatial steps at the spatial coordinates x, y, and z, respectively:

$$M_1 = [M_z^n(I,J,K)]^2 + [M_y^n(I,J,K)]^2 \qquad (4)$$

$$M_2 = \frac{1}{T_2^n(I,J,K)} M_1 \qquad (5)$$

$$M_3 = \frac{1}{[T_2^n(I,J,K)]^2} M_1 \qquad (6)$$

$$M_4 = [\gamma^n(I,J,K) M_y^n(I,J,K)]^2 \qquad (7)$$

$$M_5 = [\gamma^n(I,J,K) M_x^n(I,J,K)]^2 \tag{8}$$

$$M_6 = [\gamma^n(I,J,K) M_z^n(I,J,K)]^2 - 2[\gamma^n(I,J,K) M_y^n(I,J,K)]^2 \tag{9}$$

$$M_7 = \gamma^n(I,J,K) M_y^n(I,J,K) M_z^n(I,J,K) \tag{10}$$

$$M_8 = \gamma^n(I,J,K) M_y^n(I,J,K) M_y^n(I,J,K) \tag{11}$$

$$M_9 = \gamma^n(I,J,K) M_y^n(I,J,K)[\frac{M_z^n(I,J,K) - M_0}{T_1^n(I,J,K)} + 4\frac{M_z^n(I,J,K)}{T_1^n(I,J,K)} + 2M_0 \frac{M_y^n(I,J,K)}{T_1^n(I,J,K)}] \tag{12}$$

$$M_{10} = [\gamma^n(I,J,K)]^2 M_x^n(I,J,K) M_z^n(I,J,K) \tag{13}$$

$$M_{11} = \gamma^n(I,J,K) M_x^n(I,J,K) M_y^n(I,J,K) \tag{14}$$

The coefficient matrix $[Q]_{11\times 11}$ is defined below:

$$[Q]_{11\times 11} = [\,Q_0 \quad Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5 \quad Q_6 \quad Q_7 \quad Q_8 \quad Q_9 \quad Q_{10}\,]^T \tag{15}$$

$$Q_i = \left[Q_{i1} \quad Q_{i2}\right], \quad \text{where i=0, 1, 2, …, 9, 10} \tag{16}$$

$$Q_{i1} = [\,1 \quad -2\Delta t_i \quad (\Delta t_i)^2 \quad (\Delta t_i)^2 (B_0 - B_{1i})^2 \quad (\Delta t_i)^2 (B_0 - B_{1i}) \quad (\Delta t_i)^2 (B_{1i})^2 \quad -2\Delta t_i B_{1i}\,] \tag{17}$$

$$Q_{i2} = \left[-2(\Delta t_i)^2 (B_{1i})^2 \quad (\Delta t_i)^2 B_{1i} \quad -2(\Delta t_i)^2 B_{1i}(B_0 - B_{1i}) \quad 2(\Delta t_i)(B_0 - B_{1i})(B_0 - B_{1i} - 2)\right] \tag{18}$$

The matrix $[S]_{11\times 1}$ at the right-hand side of (3) is defined below, where $S^n(I,J,K)$ is the MR signal in rotating reference system:

$$[S]_{11\times 1} = \frac{1}{\Delta y} \mathrm{diag}(\Delta x_i)\left\{s_i^n(\Delta_i, J, K_0)] - s_i^n(0, J, K_0)\right\} \tag{19}$$

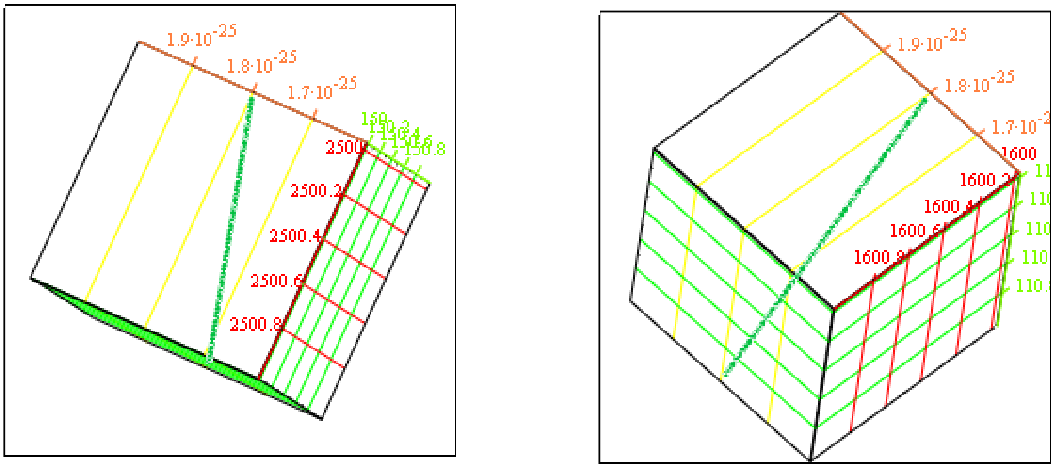
$$s_i^n(\Delta_i, J, K_0) = S^n(I+1, J+1, K_0) - S^n(I+1, J, K_0), \quad \text{where } \Delta x = \Delta x_i \text{ for i=1, 2, 3. …, 10, 11} \tag{20}$$

$$s_i^n(0, J, K_0) = S^n(I, J+1, K_0) - S^n(I, J, K_0), \quad \text{where } \Delta x = \Delta x_i \text{ for i=1, 2, 3. …, 10, 11} \tag{21}$$

Here K_0 illustrates the plane $z = z_0$. The diag defines a diagonal matrix. The Δy and Δx defines the spatial differences on the spatial coordinates y and x, respectively. Equations (13)-(21) are written for $t_0 = 0$ for brevity.

Equations (19)-(21) involve 24 different quantities, which will be measured on 24 different locations at the same time. The solution of (3) gives the M_i, where i=1 2, 3, …, 11. The solutions for γ, T_1, T_2, and magnetizetion are obtained by using M_i in (4)-(14). So, if we get one measure at a second then 24 seconds will be enough to diagnose about the organ by scanning 24 different points, only.

Various computational results are obtained related with healty organs and/or organs involving tumor [7] and some of them are depicted in Fig. 1, where B_0 =1 T, B_1 =0.2 T, M_0 =3.1×10^{-3}, ω_1=1000 s^{-1}, $\gamma = 2.675\times 10^8$ rad/(Ts).



(T1, T2, S) (T1, T2, S)

(a) (b)

Fig. 1. The variations of T_1 and T_2 with S for brain fluid: (a) normal case, (b) involving tumor case. T_1, T_2, and S are illustrated with red, green, and orange colors, respectively.

CONCLUSIONS

Some results on the studying to solve the Bloch equations in rotating reference system by applying the FDTD method. The FDTD structure converted into an appropriate inverse problem in time domain. The solutions and variations of several MRI processes with respect to the specific parameters are displayed. These specific parameters are related to diagnose an illness. Various computational results related with several healthy and/or ill organs and/or conditions are given. This study provides the possibility of scanning smaller region then the region required in common MRI applications in use; beside, provides the possibility of spending lesser time then the time spent already.

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