Coding For MIMO Wireless Systems

Erik Stauffer\(^{(1)}\), Arogyaswami Paulraj\(^{(2)}\)

\(^{(1)}\)Department of Electrical Engineering, Information Systems Laboratory, Stanford University, 228 Packard, 350 Serra Mall, Stanford, Ca 94305 USA, erstauff@stanford.edu

\(^{(2)}\)As (1) above, but Email: apaulraj@stanford.edu

ABSTRACT

MIMO wireless systems have received a great deal of attention recently, in part due to the promise of increased throughput, extended range, and improved reliability. Achieving the gains of the MIMO channel requires coding that can take advantage of these channel resources. This work studies the performance of different concatenated space-time coding and outer channel coding and develops some practical guidelines for coding design of MIMO wireless systems.

INTRODUCTION

Multiple antenna systems have received a significant amount of interest by the research community in recent years. This is mainly due to the promise of increased channel capacity, reliability, and range of these multiple input multiple output (MIMO) systems. Two main categories of available gain include diversity gain, which characterizes the reliability of the system with respect to fading, and multiplexing gain which characterizes the capacity of the system. To achieve the properties offered by the wireless channel, proper coding is required, which has led to the development of space time codes.

The design of a practical system adds several important design aspects. First, practical designs occur at finite SNR, so the asymptotic nature of diversity and multiplexing must be handled in light of other practical design considerations including coding gain and implementation complexity. Secondly, a wireless system rarely uses only a space time code, but instead utilizes a channel code such as a convolutional, turbo, or LDPC outer code. The use of these additional codes supplies additional coding performance, so it necessary to consider the performance possibilities and limitations of the concatenation of all the coding in a system.

SYSTEM MODEL

This paper considers a quasi-static block fading MIMO channel model with \(M_t\) transmit antennas and \(M_r\) receive antennas

\[
y = Hx + n \tag{1}
\]

where \(x \in \mathbb{C}^{M_t \times 1}\) is the input vector, \(y \in \mathbb{C}^{M_r \times 1}\) is the output vector, \(n \in \mathbb{C}^{M_r \times 1}\) is an additive zero mean circularly symmetric complex Gaussian noise vector distributed as \(N(0, \sigma^2 I)\), and \(H \in \mathbb{C}^{M_r \times M_t}\) is a random channel matrix, where each element is an independent circularly symmetric complex Gaussian random variable with variance one.

Concatenated channel coding and space time coding will be considered as a single block code. The receiver consists of a spatial demapper followed by a series of decoders, between which iterations are possible in general.

DEFINITIONS

Code Rate

The code rate \(R_c\) is defined as the ratio of the number of information bits and the number of bits required to describe all possible output sequences, as in [1]. For example, given a modulation of order \(|S|\), where \(|S|\) is the number of constellation points per antenna per symbol period, that is used over \(M_t\) transmit antennas and \(N\) symbol periods to transmit a total of \(m\) information bits, the code rate is defined as

\[
R_c = \frac{m}{\log_2(|S|^{NM_t})}.
\]

With this formulation, the code rate describes the entire concatenated system, including space time coding and channel coding, and can vary such that \(0 < R_c \leq 1\). The spatial rate of the code, \(R_s\), describes the number of unique symbols transmitted per symbol period, and therefore describes the contribution of any repetition coding or zero filling, and contributes to \(R_c\) in addition to any other coding.
Diversity and Multiplexing Gains

Diversity and multiplexing will be defined following the formulation of [2]. First, multiplexing gain describes a system's ability to increase throughput asymptotically with SNR, where $0 \leq r \leq \min(M_t,M_r)$.

$$\lim_{\text{SNR} \to \infty} \frac{R_t(\text{SNR})}{\log(\text{SNR})} = r$$

(2)

Similarly, diversity gain describes how the probability of error decreases asymptotically with SNR, as defined below, where $0 \leq d \leq M_t M_r$.

$$\lim_{\text{SNR} \to \infty} \frac{\log P_{\text{out}}(\text{SNR})}{\log \text{SNR}} = -d$$

(3)

CODE RATE - DIVERSITY - MULTIPLEXING TRADEOFF

Performance Tradeoff

Two main asymptotic tradeoffs regarding space time coding for MIMO systems have been identified in the literature. First is the tradeoff between multiplexing and diversity gains [2]. This tradeoff states that a given code can achieve the full multiplexing gain and the full diversity gain, and the tradeoff describes how incremental SNR can be used to either increase throughput or reduce probability of error. Secondly, there is a tradeoff between diversity gain and the code rate [3], and indicates that as the code rate goes up, less redundancy and therefore less transmit diversity is possible in a code.

Recently, these two key tradeoffs involved in coding for MIMO systems have been generalized in [1]. This tradeoff identifies that for a code to achieve the multiplexing diversity tradeoff frontier, the code rate $R_c$ must be less than $\frac{1}{M_t}$. In addition, if a code has a higher rate, then a more constrained tradeoff exists, as shown in Fig. 1. The code rate relates throughput with receiver complexity, since complexity is in general exponentially related to the number of codebits transmitted per symbol.

Achieving the Performance Surface

Codes that achieve subsections of the generalized asymptotic performance surface are known. For codes with $R_c \leq \frac{1}{M_t}$, several codes that achieve the multiplexing diversity tradeoff have been proposed, such as Lattice Codes or LAST codes [4]. Other codes that achieve this region of the performance surface for a specific number of transmit antennas also exist, such as the Yao and Wornell rotation code [5]. For the specific case of $R_c = 1$, VBLAST space time coding utilizing a ML receiver achieves the generalized frontier, but is of course suboptimal for general $R_c$ [2]. Finally, for $r = 0$ codes that achieve some portion of the code rate diversity tradeoff have been developed. Rank-distance space-time codes have been shown to achieve the frontier for all $0 < R_c \leq 1$ and $r = 0$ developed in [3] for BPSK and QPSK modulation.

PRACTICAL CODING

Design of practical concatenated coding systems presents several additional design challenges. First is that of finite SNR, which results in diminishing returns due diversity and multiplexing gain. Secondly, there are complexity concerns. As spectral efficiencies become higher, receiver design becomes increasingly complex due to the large number of constellation points per symbol and the number of transmit antennas. This is because ideal ML or MAP symbol detection complexity grows as $O(2^b M_t)$, where $b$ is the number of information bits per symbol, with the exception of some linear codes discussed in the next section. Finally, coding gain is ultimately the most important resource from the channel, in that regardless of the asymptotic properties of a code, the code with the smallest packet error rate (PER) for the target transmission rate or similarly the code with the largest throughput for a target PER will be chosen.

Linear Space Time Codes

As discussed earlier, to achieve transmit diversity, low rate coding is required. In most cases, this results in increased complexity. While capacity lossy, for low SNRs repetition coding can be shown to be close to optimal. Optimal detection of repeated symbols, though, can in some cases lead to linear receivers that are simpler than the complexity trend discussed above, such as the Alamouti code [6] and OSTBC’s [7]. These schemes are
increasingly capacity lossy at higher SNR, though, since these codes have limited multiplexing gain as discussed in [2].

Receivers

Due to the large spectral efficiencies and spatial interference, the constellation demapper’s (symbol demapper) complexity can be difficult to implement. Because of this, several suboptimal receivers have been developed. One important class of receiver is that of linear receivers, including ZF and MMSE, which decouples spatial interference. While reducing the decode complexity, these receivers suffer a loss in receive diversity. It has been shown that these receivers achieve a receive diversity order of $M_r - M_t + 1$ instead of the usual $M_r$ [8]. Block forms of these receivers are also possible. Alternative approximate nonlinear receivers have also been considered. One of these is the sphere decoder, which can implement an exact ML detection and can be much simpler than brute force ML detection. One drawback, though, is that the actual complexity depends on system parameters such as SNR, and a lower bound on the average complexity in [9] has been shown to be exponential. One variant of the sphere decoder is the sphere list decoder which allows for some soft information by using the MAP algorithm on the decoded list [10]. Another possibility is that of successive cancellation and approximate iterative soft successive cancellation [11]. With these techniques, the effect of decoded or partially decoded bits can be removed from the received signal in further iterations of the symbol demapper.

Channel Coding

Most practical wireless communication systems utilize channel codes for the available coding gain such as convolutional, turbo, or LDPC codes. Notice that the code rate defined in the generalized tradeoff isn’t limited to just that of the space time codes, but includes all concatenated coding. Therefore, we can use this surface to upper bound performance of concatenated space time and channel coding systems utilized by practical systems. Since typical channel coding rates used include 1/2 and 1/3, it is possible that the outer code can provide some additional diversity if the space-time code was originally diversity suboptimal.

An additional design facet at practical SNR’s is due to the use of channel codes with random internal interleavers such as parallel concatenated turbo and LDPC codes. While these codes have a relatively poor minimum distance $d_{\text{min}}$, and consequently relatively poor performance at very low PER, they achieve competitive coding gains at practical SNRs due to their minimum distance spectra. While the minimum distance of the code may be small, its multiplicity is made small with increasing block size and spectral thinning [12]. Similarly with fading channels, it is possible for a code with a smaller minimum codeword difference rank to perform better at practical SNRs, before the asymptotic behavior the minimum codeword difference rank dominates the performance.

Coding for Outage Capacity

Consider a 2x2 MIMO system and a design goal to approach the 10% outage capacity with one or more space time codes and outer codes, while minimizing complexity. Notice that since the outage level has been set, diversity can now be interpreted as a coding gain, though it diminishes with increasing diversity order.

Ideally for performance, we would like to use a $R_c \leq 1/2$ code with an ML receiver, but this can be computationally prohibitive, especially at higher spectral efficiencies. By relaxing our performance constraint we can simplify the implementation. For low SNRs, we can choose a code with $r = 1$ without a significant loss from the outage capacity, such as the Alamouti scheme, requiring only linear processing at the receiver and achieving fourth order diversity. Since the space time code already achieves fourth order diversity, any outer coding cannot increase the diversity order on this channel, but can only add coding gain. Neglecting any gap to capacity of this outer code, the optimal performance of this strategy is shown in Fig. 2. Due to the low multiplexing gain of this code, though, at some SNR this code will begin to diverge from the outage capacity. At this SNR, we can then switch to a $r = 2$ spatial multiplexing code. Transmit diversity can be regained with the addition of a $R_c = 1/2$ outer code, which may have already been in place for the previous $r = 1$ code. Receiver complexity can be reduced by implementing a MMSE receiver, since optimal ML detection requires joint detection of two different symbols. Following the discussion above, this scheme is limited to achieving at most second order diversity, and the performance of this strategy is shown in Fig. 2 neglecting any coding gap. Note the effective coding gain loss due to the reduced diversity order. Optimal performance with an ML receiver would correspond to the outage capacity curve shown.
Figure 1: Performance surface for a $M_t = 2$ and $M_r = 2$ MIMO system.

Figure 2: 10% Outage Performance for a 2x2 MIMO system.

CONCLUSION

Diversity and multiplexing are two key channel resources in a MIMO system. Utilizing these channel resources requires proper channel coding in order to extract these resources. Implementing this coding, though, requires computational complexity. Additionally, these channel resources are asymptotic in nature, and result in diminishing returns at practical signal to noise ratios. Finally, a practical system usually utilizes an outer code, in addition to space time coding, which also contributes to the overall performance of the system.

References