A NEW METHOD OF CHAOTIC CODE QUANTIZATION FOR THE GENERATION OF EFFICIENT CDMA CODE SETS

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***Abstract*— In this paper, we propose a new method for the selection of quantization threshold value for chaotic maps based on the statistics of the map itself and not on the statistics of the individual generated codes. The new method is based selecting the invariant probability density function (PDF) median value as a threshold value, which is independent of the used initial conditions. Computer simulations are used to evaluate the balance of the generated codes. Chaotic codes generated using this method are found to have better performance in Asynchronous CDMA Applications than codes generated using zero-threshold method.**

***Keywords*— Binary chaotic codes, Chaotic DS-CDMA , Chaotic codes quantization**

I. INTRODUCTION

Code Division Multiple-Access (CDMA) has been considered as a key technology for the third and next generation mobile communication systems. In CDMA systems, the selection of the spreading code sets plays a vital role in the system performance. Recently, a growing interest in the use of chaotic spreading sequences in CDMA systems was observed in the literature [1,2]. Among all the proposed techniques for the use of chaos theory in communications engineering, chaotic spreading codes for DS-CDMA systems are believed to be promising in terms of improving the system performance [3].

There are three identities of chaotic system. It is deterministic, it exhibits behavior that is difficult to distinguish from random behavior, and it is sensitive to initial conditions. In DS-CDMA system the spreading code has to be deterministic to make the receiver able to be synchronized with the transmitter, pseudo random code in order to make it difficult to predict the transmitted data by an undesired user and there must exist a large number of codes to be able to cope with the increase of the number of active users. From the above three properties of chaotic systems we can easily link between chaos systems and how it can be used to generate spreading codes for DS-CDMA applications [4]. The sensitivity to initial conditions is a very important property that gives us a large number of codes with respect to any known spreading code families.

Chaotic codes continuous real numbers but for most of CDMA systems binary code sets are needed. Therefore, there is a need to convert the continuous chaotic code to a quantized code by using a proper quantization function. For the generation of chaotic binary codes, a simple method is to select the threshold value to be the middle of the map output range, e.g. zero for [-1, +1] map range. This selection of threshold level may generate imbalanced binary codes. However, balancing, i.e. number of ones equal number of zeroes, is one of the important requirements of PN spreading codes. In [5], the statistics, either the code mean or median value, of each generated code is individually used to set the proper quantization threshold value. This method is not practical to be used in a CDMA system as each code in the spreading code set has a different threshold value and thus they must be stored and transmitted individually resulting in a useless overhead.

In this paper, we propose a new method for the selection of quantization threshold values for chaotic maps based on the statistics of the map itself and not on the statistics of the individual generated codes. The new method is based on noticing that each chaotic map is characterized by an invariant probability density function (PDF) that is independent of the used initial conditions and depends only on the map parameters [5]. Our approach is to use of the median value of the invariant PDF function as the quantization threshold. This selection is fixed for all codes generated from the same map with the same parameters and assures, on the average, the balance of the generated codes.

II. CHAOTIC BINARY CODE GENERATION

Consider the P-map defined (1). This map has one control parameter p. Fig. 1 shows an example of P-map with p=100.

$$X_{n+1} = \cos(px_n^2 \cos(x_n)) \qquad (1)$$

For DS-CDMA systems usually a binary, quantized version of the continuous-value chaotic sequences, codes are used. Binary chaotic code can be generated by choosing initial condition x_0, then iterating the map N times to generate a continuous-value code

$x=\{x_1, x_2, \ldots, x_N\}$. Using proper quantization function Q(x) the code ensembles $a=\{a_1, a_2 \ldots, a_N\}$ can be generated. The quantization function takes the form (2) where l_{th} is the threshold value.

$$Q(x)=\begin{cases}-1 & x<l_{th} \\ 1 & x\geq l_{th}\end{cases} \tag{2}$$

Threshold value can be simply the middle of the map range or the mean of the chaotic sequence. Each chaotic sequence has its own mean value and then has its own mean. This means if the mean value of the sequence is used as threshold value then it must be computed for each generated code which reflects an increased complexity in the system. An alternative way is to use the map characteristic to select the threshold value. In this paper we propose the use of the map invariant PDF median as a threshold value. The invariant PDF is unique for each map and describes the distribution the PDF of the map output after iterating the map several times starting from any initial conditions PDF. The invariant PDF of a map can be computed either analytically using Perron-Frobenius operator [6] or using imperical method. Imperical method starts by selecting a large number of initial conditions having any PDF and then iterating them many times in the map and ploting the histogram of the map output. Fig. 2. shows an example of invariant PDF of P -map for p=100. The invariant PDF of a map depends only on the map parameters and it is independent from the used initial condition.

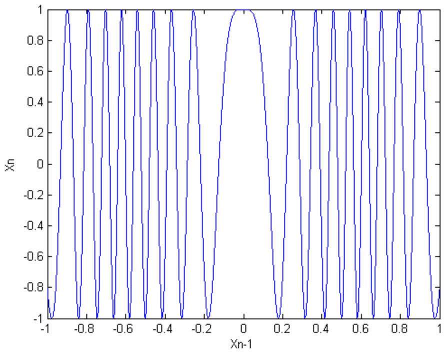

Fig. 1. P-map with p=100.

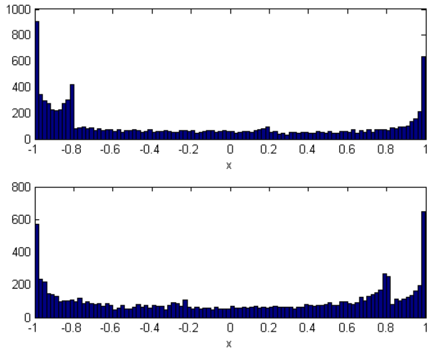

Fig. 2. P-map Invariant PDF for p=100 and p=120.

III. AN EFFICIENT METHOD OF CHAOTIC SEQUENCE QUANTIZATION

The proposed method for the quantization of chaotic codes relies on the fact that the invariant PDF of the chaotic maps depends only on the map parameters and is independent from the used initial conditions. This means that the PDF of long codes are invariant and equals to the map invariant PDF. Then instead of calculating the threshold value for each code individually depending on its PDF, the threshold value can be calculated from the map invariant PDF at first and be used for all codes generate from the same map.

We propose the use of the median value of the chaotic map invariant PDF as a threshold value. This threshold now depends on the map parameters. For symmetric maps the threshold value is always at the middle of the map range but for asymmetric maps, e.g. P-map, the threshold value differ for each value of the map parameter p.

IV. SYSTEM MODEL AND PERFORMANCE MEASURES

Consider a direct sequence asynchronous CDMA system accessed by K active users and the spreading code of the each user consists of periodic sequence of unit amplitude, positive and negative, rectangular pulses. Following the work of Pursley [7], we have (3)

$$SINR_i = \left\{\frac{R_i}{6N^3} + \frac{N_0}{2E}\right\}^{-1/2} \tag{3}$$

where $SINR_i$ is the signal to interference plus noise ratio of the i[th] user and R_i is the total interference parameter for i[th] user and given by (4)

$$R_i = \sum_{\substack{k=1 \\ k \neq i}}^{K} \sum_{l=1-N}^{N-1} C_k(l)\,[2C_i(l) + C_i(l+1)] \tag{4}$$

where $C_k(l)$ the discrete aperiodic autocorrelation function of each code and defined in (5)

$$C_k(l) = \begin{cases} \sum_{j=0}^{N-1-l} a_j^{(k)} a_{j+l}^{(k)}, & -N+1 \le l \le N-1 \\ 0, & |l| \ge N. \end{cases} \tag{5}$$

To study the effect of different threshold values on the system performance, the Performance ratio (W) measure is used (6). Performance ratio is the ratio between the Average interference parameter (AIP) of a random code set to the AIP of the code set under test.

$$W = \frac{2(K-1)N^2}{\sum_{i=1}^{K} R_i} \tag{6}$$

V. NUMERICAL EXAMPLES

Computer simulation was used to study the performance of the proposed quantization. To study the balance of the codes generated using PDF median value, the P-map invariant PDF median was calculated for different values of p (Fig. 3). Then each median value is used to quantize a chaotic code generated using P-map with the same value. For each p value the code imbalance was calculated. Those codes were also quantized using zero-threshold. Fig. 4 shows that chaotic codes quantized using the proposed method has better balance properties that those quantized using zero threshold.

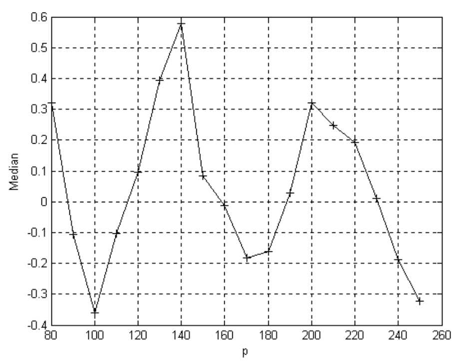

Fig. 3. P-map's Invariant PDF median for different p.

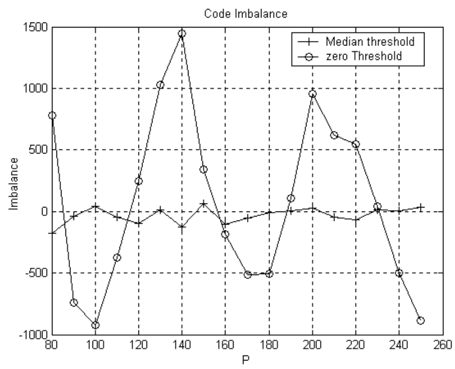

Fig. 4. Code Imbalance for zero-threshold and invariant PDF median.

In order to evaluate the performance of the codes generated using the proposed method another simulation was done. For a specific code length and code set size, 1000 code sets was generated once using the zero threshold and another using the proposed method. The AIP and the performance ratio were calculated for each code set. Average of AIP of the 1000 code set is presented in Table 1 for different code lengths and code set sizes. Fig. 4 compares between the performance ratio (W) of the quantized codes using the proposed method and using the zero-threshold method. It is clear that the proposed method performs better that the zero-threshold method or different code set sizes.

Code Set Number	Code Length	Number of Codes	AIP		AIP of Random Code
			Zero-threshold	PDF middle	
1	15	2	434	407	450
2	15	4	1,303	1,232	1,350
3	15	15	6,090	5,758	6,300
4	31	6	9,207	8,656	9,610
5	31	33	59,109	55,972	61,504
6	63	6	37,633	35,975	39,690
7	63	65	487,090	461,710	555,660
8	127	18	525,570	497,580	508,030

TABLE I. AIP of different code set sizes.

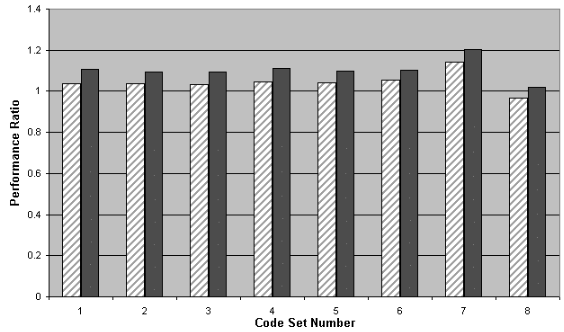

Fig. 5. Performance ratio of zero-threshold (white) and invariant PDF median (black).

VI. CONCLUSIONS

In this paper a new method for the quantization of chaotic binary codes is presented. This method depends on the generator map invariant PDF median value as the threshold for the quantization function. For the proposed method, there is no need for the calculation of the threshold value for each generated code. One threshold value is used for all the generated codes from the specific map. The code balance of the generated codes is much better that the zero-threshold method. The performance of the codes quantized by the proposed method in Asynchronous CDMA system was evaluated. Results show an increase in the system performance, in terms of minimizing Multiple Access Interference (MAI), for the codes quantized using the proposed method.

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