Low-Damping Guided Modes along Nano-Transmission Lines with Chains of Quadrupolar Resonant Plasmonic Nano-Particles

Andrea Alù[1,2], and Nader Engheta[2]
(1) University of Roma Tre, Dept. of Applied Electronics, Rome, Italy
(2) University of Pennsylvania, Dept. of Electrical and Systems Engineering, Philadelphia, PA 19104, U.S.A.

The interest in reliable and efficient "transmission-lines" as guided-wave structures in the infrared and optical regimes has considerably grown recently. The need for increasing the frequency of operation in order to reduce the dimensions of the electronic and photonic components, in fact, requires the design of new systems and structures for transporting and radiating the information at frequencies for which good electric conduction is not readily available in nature, i.e., in the infrared and optical frequencies, where metals behave as plasmonic materials.

One of the ideas proposed to overcome this problem has been to employ linear chains of resonant plasmonic nano-particles [1-6]. Under suitable conditions, in fact, it has been shown experimentally [1, 2] and theoretically [3-6] how these arrays can guide energy with a modal cross section much smaller than the free–space wavelength, and with a sufficiently low damping factor. We have been interested in finding the conditions under which this technique can be successfully utilized in the case of arrays of resonant plasmonic nano-spheres and the properties of such guidance with respect to the parameters that come into play [6]. In all the cases reported in the literature, however, the analysis has been limited to only dipolar resonances, i.e., those dominant resonances for which the radiated field from a single nano-structure has a dipolar form. As it is well known, in the case of a homogeneous nano-sphere such resonances happen at frequencies for which $\varepsilon \simeq -2\,\varepsilon_0$, where ε is the sphere's material permittivity and ε_0 is the permittivity of free space. In this case, the electric polarizability of each sphere $\alpha_{ee} \simeq 4\pi\varepsilon_0 a^3 \dfrac{\varepsilon-\varepsilon_0}{\varepsilon+2\varepsilon_0}$ (with a being the sphere radius) attains a large value. Fig. 1 (top panel) shows a sketch of the configuration of induced longitudinal dipoles in such an array for a horizontal excitation. It can be shown in this case that the dispersion relation for the natural eigen-modes of such a chain is represented by the following condition:

$$\mathrm{Li}_3\left(e^{-j\left(\bar\beta+1\right)\bar d}\right)+\mathrm{Li}_3\left(e^{j\left(\bar\beta-1\right)\bar d}\right)+j\bar d\left[\mathrm{Li}_2\left(e^{-j2\pi\left(\bar\beta+1\right)\bar d}\right)+\mathrm{Li}_2\left(e^{j\left(\bar\beta-1\right)\bar d}\right)\right]=\frac{2\pi\varepsilon_0\bar d^3}{\bar\alpha},$$

where $\mathrm{Li}_n\left(x\right)$ is the polylogarithm function of order n [7], and $\bar d = k_0 d$, $\bar\alpha = k_0^3\alpha_{ee}$, k_0 is the free space wave number and d is the centre-to-centre distance between two neighboring spheres. In the above expression the natural mode is supposed to propagate along the z axis (the axis of the chain) with factor $e^{-j\bar\beta k_0 z}$, where dimensionless $\bar\beta$ in general may be complex.

In the limit of no material losses in the spheres, this dispersion relation admits no-damping solution (i.e., $\bar\beta$ is real), under the condition:

$$\sum_{n=1}^{\infty}\frac{(-1)^n\left[\cos\left(n\bar d\right)+n\bar d\sin\left(n\bar d\right)\right]}{n^3\bar d^3}<\pi\varepsilon_0\,\mathrm{Re}\,\bar\alpha^{-1}<\sum_{n=1}^{\infty}\frac{\cos\left(n\bar d\right)\left[\cos\left(n\bar d\right)+n\bar d\sin\left(n\bar d\right)\right]}{n^3\bar d^3}$$

and the further constraints $\bar d < \pi$, $1<\bar\beta<\pi/\bar d$.

Fig. 1 (bottom panel) displays the dispersion region for the zero-damping modes satisfying the above condition. This plot suggests that for a given distance between neighboring spheres along the chain (horizontal axis) the region spanned between the two curves represents the admissible values of the polarizability of each sphere (vertical axis) in order to support a propagating mode without radiation loss.

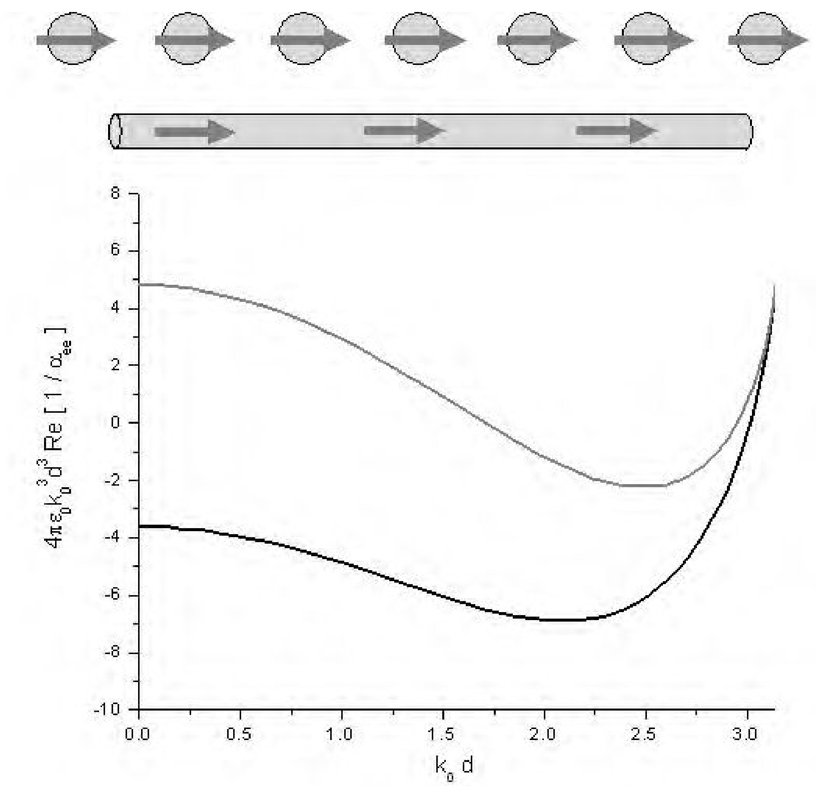

Figure 1 – Geometry (top), analogy with a wire (center) and dispersion plot (bottom) for a longitudinal dipolar chain of nano-spheres.

In a different context, in our analysis of the properties of resonant plasmonic concentric nano-shells, we have shown how, by changing the geometry of the nano-particles, and in particular by adjusting the ratio between the core-shell radii in such composite nano-spheres, it would be possible to excite not only the dipolar resonance, but also other higher-order resonances [6]. This property, which has also been observed experimentally by other groups [8], may in principle be exploited in the design of a more efficient nano-transmission line in this problem. One may indeed notice how the geometry of Fig. 1 and its induced dipole moments on the spheres may resemble the case of a simple wire carrying an electric current. In the chain of spheres the “displacement current” is the one that flows, analogous to the electric conduction current carried by a standard wire at lower frequencies. This analogy is sketched in Fig. 1 (centre). On the other hand, when one can induce a quadrupolar resonance in each sphere, one will expect to find a closer correspondence with a two-wire transmission-line with an odd mode, in which the electric current circulates along two wires in opposite directions, exhibiting lower radiation losses. From this heuristic analogy, we anticipate to obtain a “wider” range of permissible quadrupolar polarizability for the nanospheres in order to support zero-damping guided modes along this chain of spheres in this configuration. This heuristic analogy is depicted in Figure 2 (top and center panels).

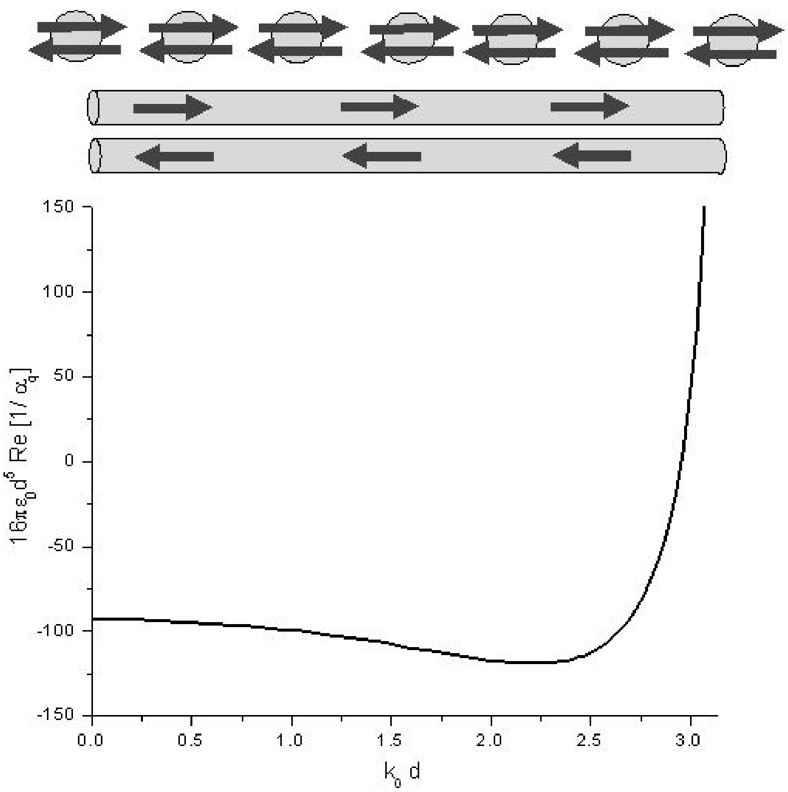

Figure 2 – Geometry (top), analogy with a two-wire transmission-line supporting an odd mode (centre) and dispersion plot (bottom) for a quadrupolar chain of nano-spheres.

The dispersion relation for the natural modes supported by this quadrupolar chain is represented by the following expression:

$$
\begin{aligned}
&48\left[\mathrm{Li}_5\left(e^{-j(\bar{\beta}+1)\bar{d}}\right)+\mathrm{Li}_5\left(e^{j(\bar{\beta}-1)\bar{d}}\right)\right]+48j\bar{d}\left[\mathrm{Li}_4\left(e^{-j(\bar{\beta}+1)\bar{d}}\right)+\mathrm{Li}_4\left(e^{j(\bar{\beta}-1)\bar{d}}\right)\right]+\\
&\quad -21\bar{d}^2\left[\mathrm{Li}_3\left(e^{-j(\bar{\beta}+1)\bar{d}}\right)+\mathrm{Li}_3\left(e^{j(\bar{\beta}-1)\bar{d}}\right)\right]-5j\bar{d}^3\left[\mathrm{Li}_2\left(e^{-j(\bar{\beta}+1)\bar{d}}\right)+\mathrm{Li}_2\left(e^{j(\bar{\beta}-1)\bar{d}}\right)\right]+\\
&\quad\quad +\bar{d}^4\left[\mathrm{Li}_1\left(e^{-j(\bar{\beta}+1)\bar{d}}\right)+\mathrm{Li}_1\left(e^{j(\bar{\beta}-1)\bar{d}}\right)\right]=\frac{16\pi\varepsilon_0\bar{d}^5}{\bar{\alpha}_q}
\end{aligned}
$$

where $\bar{\alpha}_q \equiv k_0^5\alpha_q$ and α_q is the quadrupolar polarizability (i.e., quadrupolarizibility) of the particles, which by definition relates the induced quadrupole moment $\underline{\mathbf{Q}}$ to the symmetric part of the gradient of the impinging electric field: $\underline{\mathbf{Q}} = \alpha_q\left(\nabla\mathbf{E}+\mathbf{E}\nabla\right)/2$.

It is interesting to note that this expression reflects the fact that the contributions from more distant nano-spheres (related to the first order polylogartihm functions) and the from the nearest-neighbor nano-spheres (related to the higher order polylogarithms) weigh more in this case than in the longitudinal dipolar case, which is consistent with the heuristic prediction that the mode should be "more guided" along the chain. In this case the condition for having zero-damping guided modes becomes:

$$
\pi\varepsilon_0\,\mathrm{Re}\frac{1}{\bar{\alpha}_q} > \sum_{n=1}^{\infty}\frac{(-1)^n\left[\cos\left(n\bar{d}\right)\left[6-\frac{21}{8}n^2\bar{d}^2+\frac{1}{8}n^4\bar{d}^4\right]+\sin\left(n\bar{d}\right)\left[6n\bar{d}-\frac{5}{8}n^3\bar{d}^3\right]\right]}{n^5\bar{d}^5},
$$

with the same additional constraints of $\bar{d} < \pi$, $1 < \bar{\beta} < \pi / \bar{d}$. The dispersion plot is shown in Fig. 2 (bottom panel).

The permissible range of the normalized quadrupolarizability along the vertical axis in this case does not have any upper bound, consistently with the above inequality. It is evident therefore how the quadrupolar chain allows a broader dispersion for the zero-damping propagating guided modes, when compared with the dipolar chains.

In the talk we will provide a general analysis of the radiating and guided mode characteristics of such quadrupolar chains, providing design examples and physical insights into this phenomenon, and comparing these results with corresponding guided modes for dipolar chains.

ACKNOWLEDGEMENTS

This work is supported in part by the U.S. Defense Advanced Research Projects Agency (DARPA) Grant Number HR0011-04-P-0042. Andrea Alù has been partially supported by the 2004 SUMMA Graduate Fellowship in Advanced Electromagnetics

REFERENCES

[1] S. A. Maier, P. G. Kik, and H. A. Atwater, "Optical pulse propagation in metal nanoparticle chain waveguides," *Physical Review B*, vol. 67, 205402, May 6, 2003.

[2] M. Quinten, A. Leitner, J. R. Krenn, F. R. Aussenegg, "Electromagnetic energy transport via linear chains of silver nanoparticles", *Optics Letters*, 23, 17, 1331, 1998.

[3] S. A. Tretyakov, and A. J. Vitanen, "Line of periodically arranged passive dipole scatterers," *Electrical Engineering,* vol. 82, pp. 353-361, 2000.

[4] A. D. Yaghjian, "Scattering-matrix analysis of linear periodic arrays," *IEEE Transactions on Antennas and Propagation*, vol. 50, no. 8, pp. 1050-1064, Aug. 2002.

[5] R. A. Shore and A. D. Yaghjian, "Traveling Electromagnetic Waves on Linear Periodic Arrays of Small Lossless Penetrable Spheres," *Proc. of 2004 International Symposium on Antennas and Propagation (ISAP'04)*, Sendai, Japan, pp. 425-428.

[6] A. Alù, and N. Engheta "Polarizabilities and Effective Parameters for Collection of Spherical Nano-Particles Containing Concentric Double-Negative or Single-Negative Shells," in *Proc. of 2004 URSI International Symposium on Electromagnetic Theory*, Pisa, Italy, pp. 24-26, May 23-27, 2004. Extended full-length manuscript has been submitted, under review, (the manuscript can be viewed online at http://arxiv.org/pdf/physics/0410011).

[7] L. Lewin, *Polylogarithms and Associated Functions*, New York: North-Holland, 1981.

[8] S. J. Oldenburg, G. D. Hale, J. B. Jackson, and N. J. Halas, "Light scattering from dipole and quadrupole nanoshell antennas," *Applied Physics Letters*, vol. 75, no. 8, pp. 1063-1065, Aug. 23, 1999.