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A Novel Lightning Strike Location Prediction Method

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Abstract

We describe a new methodology of calculating the electrostatic field enhancements in the near-field of structures. This method is then used to calculate the fields near buildings of several different heights. We compare the relative enhancements with the heights of those buildings and several models which attempt to predict lightning strike occurrences to buildings as functions of their height. It is shown that our model matches results from data of lightning strikes to isolated buildings vs. their heights. We take our model further to discuss the effect that several buildings surrounding one another has on these solutions and the likelihood of strikes to each structure.

1 Introduction

The development of a lightning leader and its path of propagation are largely determined by the shape and magnitude of the electric field between the storm-cloud and the surface of the earth [1]. For this reason, the solution to Laplace's equation in the space between these locations is a desired result. Unfortunately, numerical techniques for the solution to this class of problems are often cumbersome, inaccurate, or take a very long time to compute. It is for this reason that researchers will often utilize statistical approaches to generate empirical equations which map parameters like building height, and lightning strike density to the expected number of strikes to a structure per year [2]. We present in this paper a novel methodology of solving for the electrostatic field enhancements in the vicinity of structures using a boundary relaxation technique [3] that allows for the solution of the fields around these structures as though they were in free space and above an infinite and perfect-conducting ground-plane. We use this solution to derive a function which relates building height to expected number of lightning strikes. We then use this tool to investigate the mutual coupling effects between adjacent structures.

Section 1 of this paper will describe the modeling methodology. Section 2 will use the model to investigate the relationship between isolated building structure heights and the expected number of strikes to those buildings, in which we compare our results with other empirically-derived equations and data. Section 3 will discuss further applications of this model applied across cityscapes composed of several adjacent structures whose mutual field coupling degrades the accuracy of previous empirical calculations.

2 The Numerical Solution to Laplace's Equation Using Boundary Relaxation

The electric potential in the vicinity of a conductor may be found via the solution to Laplace's equation

$$\nabla^2\Phi = 0 \tag{1}$$

and from there the electric field may be found as

$$\vec{\boldsymbol{E}} = -\nabla\Phi \tag{2}$$

There are many numerical techniques for the solution of eq. 1 which are well-known [4]. The general form of each starts by discretizing the solution space into a set of numerical voxels, each of which are defined by a location, some physical properties, and perhaps a potential of their own, then defining the appropriate boundary conditions on the device-under-test (DUT) and the grid space boundary itself. After that, all of these numerical values are put into a matrix and that matrix is solved iteratively over the numerical form of eq. 1, which is

$$\begin{aligned}&\frac{\Phi(i+1,j,k)+\Phi(i-1,j,k)+\Phi(i,j+1,k)}{6}\\ +&\frac{\Phi(i,j-1,k)+\Phi(i,j,k+1)+\Phi(i,j,k-1)}{6}\\ &\qquad\qquad\qquad\qquad\qquad = \Phi(i,j,k)\end{aligned} \tag{3}$$

in a simple geometric case, where the discrete points (i, j, k) correspond to the cartesian coordinates (x, y, z). Over the course of the iterative solution, the potential value at the DUT is held to the electric potential boundary condition, typically

$$\Phi(\vec{\boldsymbol{r}}') = 0 \tag{4}$$

where $\vec{\boldsymbol{r}}'$ is the location of the conductor.

The proper treatment of the grid space boundary condition is often a source of complexity or error, especially when these solutions are supposed to represent the field around some object in open space. Recall that the analytical solution to these open space problems takes eq. 4 and $\Phi(\vec{\boldsymbol{r}} \to \infty) = 0$, where $\vec{\boldsymbol{r}}$ is the observation point. In a computer, however, we are not capable of defining our solution space out to infinity and so we are left to define a boundary condition which must be held fixed on the grid space boundary, and this translates physically to the grid space boundary acting as a conductor.

One of the simpler techniques employed to improve this error is to move the grid boundary further and further away from the DUT. Each expansion of the grid space has the desired effect of lowering the error incurred by the finite grid space, but the solution computation time grows proportionally to the cube of this grid space expansion and the answer is still wrong regardless of how far one may stretch the space. Annulus methods such as ballooning, the infinite/infinitesimal elements methods, and various spatial transforms are other iterative-type solutions. There are also non-iterative forms such as asymptotic boundary conditions (ABC) and the measured equation of invariance (MEI) approach. An in depth discussion of each is beyond the scope of this paper, but are found in the review by [5]. The majority of these solutions are purely numerical, require lengthy computation times, may not converge at all, and are often still approximations. We solve this problem by using a boundary relaxation technique [3].

Assume we have solved Laplace's equation around some DUT in an otherwise uniform, vertical electric field, and have not attempted to deal in any way with the grid space boundary potential; we simply hold the potential on the grid space boundary at the uniform, vertical field potential. Holding this potential fixed physically translates to this location in the simulation space functioning as though it were itself a conductor. This means that the surface charge density produced on the DUT as a result of the uniform electric field will produce an image charge density on the grid space boundary. We use the surface charge density on the conductor to subtract away the image charge density on the grid boundary. After doing so, we run through the solution of Laplace's equation again with our newly calculated grid space boundary condition. We repeat this process several times until virtually no image charge is produced and the solution has converged on the free space solution.

The method by which we remove the image charge density uses the potential from the charge density on the DUT, ρ_s. After solving Laplace's equation once, we calculate ρ_s, and from there we calculate the potential ρ_s produces at the grid boundary by

$$\Phi_\Delta = \frac{1}{4\pi\varepsilon_0}\int_{S'} \frac{\rho_s}{|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'|} dS' \tag{5}$$

where ε_0 is the permittivity of free space, and S' is the DUT surface. We add that to initial grid boundary condition, such that the new grid boundary potential is

$$\Phi_{n+1} = \Phi_n + \Phi_\Delta \tag{6}$$

where Φ_n is the initial grid boundary condition and Φ_{n+1} is the grid boundary condition for the next Laplace iteration which has discounted evermore image charge density.

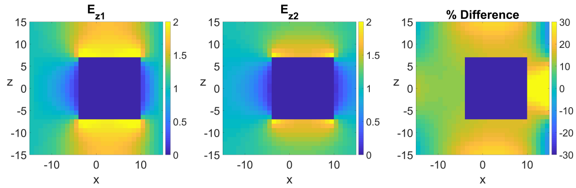

Figure 1. Left: The solution to Laplace's around a cube. Center: The same solution using boundary relaxation. Right: The percent difference of the two solutions.

Although this method does require the solution to Laplace's equation several times, with 5 total boundary relaxation iterations appearing to be the ideal amount, we are able to move the grid boundary arbitrarily close to the DUT, providing at least a few grid cells for the update equations in eq. 3. Recalling that lengthening and shortening of the grid space varies the simulation time by the third power of the spatial change, it is of little consequence that we must iterate Laplace's equation several times because these multiple iterations only linearly increase the computation time, where it is already decreased by a power of three versus other methods. Iterations after the first solution also take substantially less time when previous potential values are recycled in the solution.

Figure 1 shows an example solution of Laplace's equation with no manipulation to the grid space boundary on the left (E_{z1}), 5 boundary relaxations in the middle (E_{z2}), and the percent difference between the two solutions on the right. We see on the left that the fields between the DUT and the grid space boundary are much larger on the top and bottom and much smaller on the left and right. This effect is due to the presence of the image charge. We also see that the percent difference in solution is the largest on the side of the cube closest to the grid space boundary.

3 The Relation of Lightning Strike Number to Structure Height

It is well understood that an increase in height of a structure leads to an increase in that sturcture's likelihood of being struck by lightning [6]. The increase in height leads directly to an increase in the electric field enhancements at the structure's top. The electric field between the start of the strike and its end is a crucial determinant of its strength and the path the strike will take. Given the probabilistic nature of a lightning strike happening, and its path, these enhancements only make it more likely that lightning may strike and are not an analytical predictor of when and where.

The calculation of the electric field between the cloud and structure is often a very complicated computation. It is the solution of Laplace's equation in the region, and the more accurate solutions will include distinct building features like structures on top or in the vicinity. Because of the complex nature of this computation and the many other factors leading to a lightning strike in a given location, researchers will often take an empirical approach and match data findings with a type of equation.

The researchers in [2] take this approach using data over a given region of the city of Hong Kong. They compare a data-fit equation with three other well-known equations. Those four equations are

$$
\begin{aligned}
N_{s1} &= 153\times10^{-6}N_g\times(h+15)^{4/3} && (7)\\
N_{s2} &= 640\times10^{-6}N_g\times(h)^{1.2} && (8)\\
N_{s3} &= 2106\times10^{-6}N_g\times(h)^{0.96} && (9)\\
N_{s4} &= h^{0.74} && (10)
\end{aligned}
$$

where N_g is the number of ground flashes per sq. km per year and h is the height of the building. The fourth equation is derived using the data from lightning strikes to tall buildings in Hong Kong [2], whereas the first 3 are empirically based from multiple data sets in various locations. Being that the electric field is closely related the N_s, and N_s is also a function of height, it should be the case that the trend present in electric field enhancement vs. height resembles the trend present in N_s vs. height.

We model the electric field enhancements in the presence of 5 isolated towers from 50-250m in height and each 12m in length and width, which is shown in Figure 2. We see that the field at the top of each building increases with increasing height by a substantial amount. The mean value of the electric field enhancement at the top of each building is

$$E_m(h)=\frac{1}{N}\sum_{x',y'}\sqrt{E_x(x',y',h)^2+E_y(x',y',h)^2+E_z(x',y',h)^2} \quad (11)$$

where the sum over x' and y' includes all grid cells, N, at the height, h. We now take eqs. (7-11) and divide each by their own sums as

$$
\begin{aligned}
N'_{s1} &= \frac{153\times10^{-6}N_g\times(h+15)^{4/3}}{\sum_h N_{s1}(h)} && (12)\\
& \dots && (13)\\
N'_{s5} &= \frac{E_m(h)}{\sum_h E_m(h)} && (14)
\end{aligned}
$$

so that the dependence on model parameters like N_g is removed and the resulting functions are probabilistic in nature and thus directly comparable.

Figure 3 shows the results of these calculations over the heights 50-250m in the top plot, as well as the percent error between our methodology and the 4 other equations in the bottom plot. We see that the error between our model and N_{s1}, N_{s2}, and N_{s3} suggests a fairly good fit for building heights between 150-250m, and that the match between our model and the data-driven model, N_{s4}, is a nearly perfect match with an error of less than 1% for all heights.

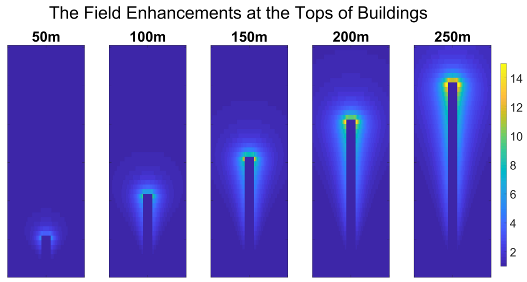

Figure 2. The field enhancements in the vicinity of five buildings of various heights.

Figure 3. Top: The number of lightning strikes expected vs. height for three general equations, one case-study equation, and our model. Bottom: The percent between each equation and our model.

This suggests that the field enhancements calculated by our model offer an incredible tool for the prediction of lightning strikes to structures in an analytical way, as opposed to empirical. Further, our model has the capability to consider very complex structure geometry with relative ease and simulations of moderate size can be run on most personal computers.

4 The Probability of Lightning Strikes to Buildings Over a City-Scape

The previous section showed that our modeling methodology is capable of predicting the relative occurrence of lightning strikes to isolated buildings as a function of their heights. A further complication arises in these techniques when structures are in close proximity to other structures of comparable height and the empirical equations of the previous section were derived in cases where buildings were largely isolated from others.

Figure 4 shows an example cityscape where 5 of the 12 buildings are the equivalent height of the isolated models of section 3, but are surrounded by other buildings of various heights, some of which have structures on top or varied widths. We show in Figure 5 the field enhancements on top

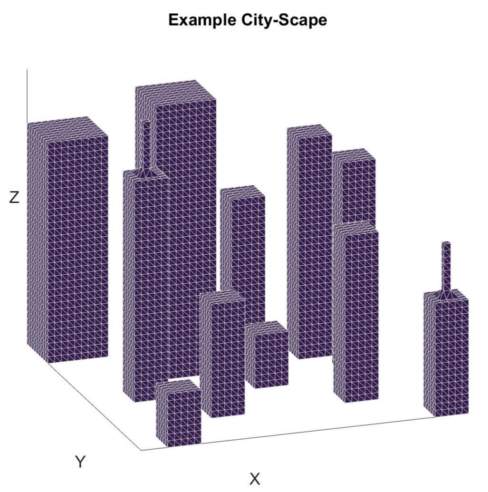

Figure 4. An example cityscape for our lightning prediction model.

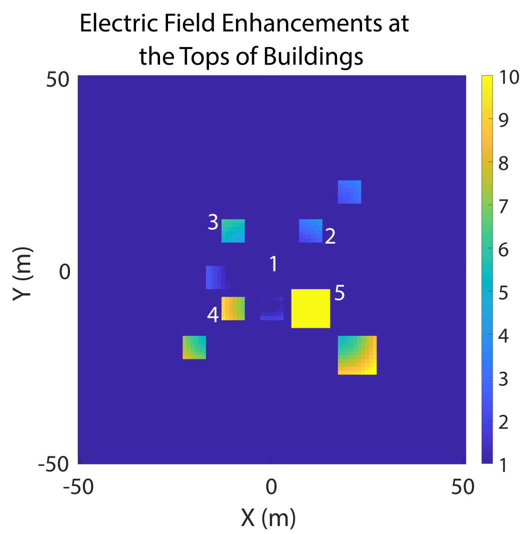

Figure 5. The field enhancements at the top of every building, with 1-5 labelled and corresponding to heights 50-250m.

of the 12 buildings, 5 of which are labelled 1-5 that correspond to buildings with heights of 50-250m. The results shown indicate that mutual coupling effects tend to minimize field enhancements in locations on buildings which are adjacent to other buildings. For instance, the structure to the southeast of building 5 has a significantly higher field enhancement in it southeast corner vs. its northwest corner. This effect is known as shielding and is well-known [7]. These results suggest that we can use our modeling tool to directly calculate this shielding effect analytically and with robust structure detail in the future.

5 Conclusion

We have described a novel methodology of solving for the electric field enhancements at the surface of conductors in a more accurate and faster way than many other methods. This methodology is used to derive a lightning strike prediction vs. height function which, when compared to other models and data, matches real-world data exceptionally well. We have proposed this model's use in the prediction of lightning strikes over a cityscape, where previous methods lack the ability to account for the mutual coupling and shielding effects between buildings of similar heights which are close together.

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