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# A Domain Decomposition Scheme with an Efficient Multitrace Multiresolution Preconditioner for the Simulation of Complex Composite Problems

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## Abstract

In this work we present a multitrace method including an automatic and multilevel quasi-Helmholtz decomposition integrated with the domain decomposition method for the solution of arbitrary complex geometries composed of piecewise homogeneous composite objects. A numerical experiment demonstrates the flexibility of the proposed approach for the solution of large multi-scale objects composed of multiple materials.

## 1 Introduction

Surface integral equation (SIE) methods based on the method of moments (MoM) [1] constitute a powerful tool in computational electromagnetics (CEM), which has become indispensable for the simulation and engineering of a wide range of applications with interest to science and industry. In this context, domain decomposition methods (DDMs) have proven to be the cornerstone for the versatility and accuracy of the SIE methodology, especially when highly complex problems come into play [2–6].

Essentially, the DDM is a preconditioner based on splitting up the original system by providing a collection of independent subdomains. Physics of different scales can be isolated into different subdomains, which can be solved locally using methods tailored to their particular features. For example, direct MoM solvers could be used for geometrically intricate but small enough subdomains, while fast iterative solvers like MLFMA [7, 8] could be applied for medium to large subdomains. This scheme greatly speeds up convergence in the case of multi-scale problems.

Nevertheless, the effective solution of increasingly complex high fidelity models as demanded by the industry must be undertaken with extreme care. It is well known that the judicious selection of the geometric partition plays a fundamental role in the performance of the method [9, 10]. In the case of large and deep multiscale problems, with several scales of geometric resolution, this partition can result in a transfer of multi-scale features to the local solvers, thus increasing their conditioning and time to solve. A representative example is the case of a large supporting structure containing large complex subsystems with multiple internal details (e.g., large cavities or large antenna arrays). In such challenging problems, the incorporation of efficient preconditioners to the local solvers is essential to improve the convergence in the local stage, which will result in the overal performance of the method, even making it possible to render correct results in the case of some challenging real-life problems.

Due to its multilevel nature, the multi-resolution (MR) preconditioner has proven to be a good choice to improve convergence in multiscale problems [11–14]. However, despite the recent efforts to bring the capabilities of this efficient physics-based preconditioner to the resolution of real-life large-scale problems [13, 14], to date its application is limited to perfect electric conductor (PEC) bodies. Notwithstanding, we have recently demonstrated that homogeneous dielectric objects can also benefit from the application of the MR preconditioner [15]. It would therefore be desirable to include this methodology in the DDM framework to accelerate convergence in solving complex realistic problems like those mentioned above in those cases including composite materials and deep multiscale features.

In this paper, the MR preconditioner is incorporated to the MLFMA-FFT solver and embedded into the SIE-DDM implementation for the solution of realistic large and deep multiscale problems involving composite materials. Considering that this preconditioner is based on a quasi-Helmholtz decomposition, which prevents its use in the presence of multimaterial junctions, a multitrace [16, 17] approach has been chosen. This will equip the local solvers with a strong ability to efficiently handle realistic medium to large subdomains dominated by multiscale features. Numerical experiments are shown that demonstrate the capabilities and versatility of the proposed methodology.

## 2 Formulation

### 2.1 The multitrace MR Preconditioner

We consider the local coupling multitrace (LCMT) formulation proposed in [17]. Let us start with the solution of a piecewise homogeneous penetrable object in a homogeneous medium. The complete object enclose $M$ different regions, denoted by $R_i$, with $i = 1..M$. Let us denote with $S_i$ the boundary surface enclosing region $R_i$, with $\hat{n}_i$ the unit vector normal to $S_i$ and pointing towards the interior of this region (see Fig. 1).

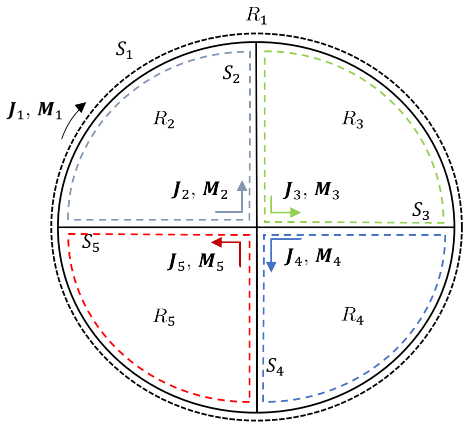


**Figure 1.** Arbitrarily shaped 3-D piecewise homogeneous object.

Let us now introduce the equivalent electric and magnetic currents $\mathbf{J}_i$ and $\mathbf{M}_i$ placed on the boundary surfaces $S_i$, where the following equivalence relations hold: $\mathbf{J}_i = \hat{n}_i \times \mathbf{H}_i$ and $\mathbf{M}_i = -\hat{n}_i \times \mathbf{E}_i$, being $\mathbf{E}_i$ and $\mathbf{H}_i$ the electric and magnetic fields in $R_i$, respectively. The so defined equivalent currents are independent in each region and naturally do not satisfy boundary conditions across interfaces between adjacent regions.

Applying the equivalence principle [1] in each region $R_i$, the electric current combined field integral equation (JCFIE$_i$) and the magnetic current combined field integral equation (MCFIE$_i$) can be derived on the boundary surfaces $S_i$ [18]. The Robin transmission conditions (RTCs) can be then applied to weakly ensure the boundary conditions across interfaces between adjacent regions as:

$$\begin{aligned} \mathbf{J}_i - \hat{n}_i \times \mathbf{M}_i + \mathbf{J}_j + \hat{n}_j \times \mathbf{M}_j = 0 \\ \hat{n}_i \times \mathbf{J}_i + \mathbf{M}_i - \hat{n}_j \times \mathbf{J}_j + \mathbf{M}_j = 0. \end{aligned} \tag{1}$$

At this point, the MR preconditioner [11, 13] is considered to improve the spectral properties of the LCMT matrix system. By virtue of the multitrace formulation outlined above, the equivalent currents in each region are discretized in terms of an independent set of basis/testing functions defined on the respective (closed) boundary surfaces $S_i$. Therefore, the MR procedure can be independently applied to the equivalent currents $\mathbf{J}_i$ and $\mathbf{M}_i$ of $R_i$ spreading along its entire boundary surface $S_i$. This is key to generating a complete set of multiresolution basis functions, including all topological loops in $R_i$.

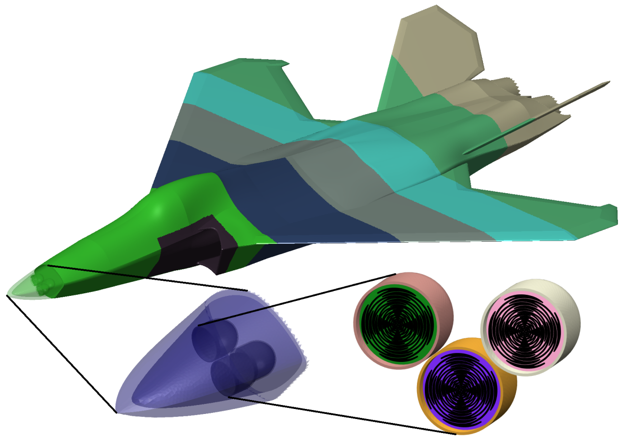

**Figure 2.** Partition into subdomains of the aircraft example. Detailed view of the radome and the 3 sinuous antennas.

## 2.2 DDM-MR method

We start with the decomposition of the original problem into a collection of $p$ subdomains ($D_i$, $i = 1, \ldots, p$) depending on the geometrical features. Then, an additive Schwarz DDM preconditioner [19] can be applied for the solution of the MoM matrix system ($\mathbf{Z} \cdot \mathbf{I} = \mathbf{V}$) as a left-preconditioner along the solutions of the individual subdomains, posing the block-diagonal matrix $\mathbf{P}$, where each diagonal block ($\mathbf{P}_i$) can be conceptually defined as $\mathbf{P}_i = \mathbf{Z}_i^{-1}$, with $\mathbf{Z}_i$ the original MoM matrix of the domain $D_i$. $\mathbf{Z}$ is the impedance matrix of the complete problem and $\mathbf{V}$ is the global excitation vector. Superscript $-1$ stands for matrix inversion.

The DDM preconditioner matrix vector product (MVP) involving the block diagonal matrix $\mathbf{P}$ can be then expressed in each subdomain as $\hat{\mathbf{I}}_i = \mathbf{P}_i \cdot \hat{\mathbf{V}}_i$, where $\hat{\mathbf{V}}_i$ is the solution (restricted to subdomain $D_i$) of the global MVP (corresponding to the mutual coupling between subdomains), and where $\mathbf{P}_i$ was defined above as the diagonal block of the preconditioner corresponding to subdomain $D_i$. This last MVP can be expressed in $D_i$ as a new independent matrix system in the form $\mathbf{Z}_i \cdot \hat{\mathbf{I}}_i = \hat{\mathbf{V}}_i$, which can be solved by the method best tailored to the characteristics of each subdomain. In our case, the MR preconditioner is applied at the local stage for the solution of those subdomains that are too large to be solved by direct solvers and keep multiscale features that slow down iterative convergence. This means solving the following systems: $\mathbf{Z}_i^{MR} \cdot \hat{\mathbf{I}}_i^{MR} = \hat{\mathbf{V}}_i^{MR}$, where the superscript $MR$ denotes expansion in terms of MR basis functions.

# 3 Numerical results

To demonstrate the versatility and efficiency of the proposed DDM-MR aproach we present a challenging numerical example consisting a realistic prototype of the near generation aircraft of the FCAS (future combat air system) program, as illustrated in Fig 2. The dimensions of the aircraft are approximately 20 m length, 16 m beam and 4 m

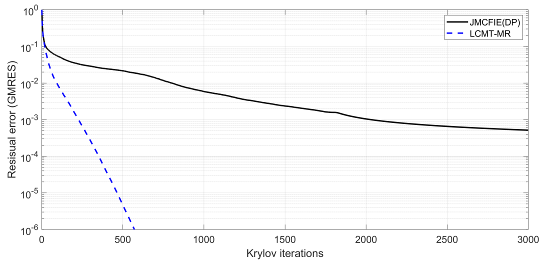


**Figure 3.** Convergence performance of the inner GMRES when solving one outer GMRES iteration of the radome domain using the proposed MR preconditioner method and the block diagonal preconditioner.

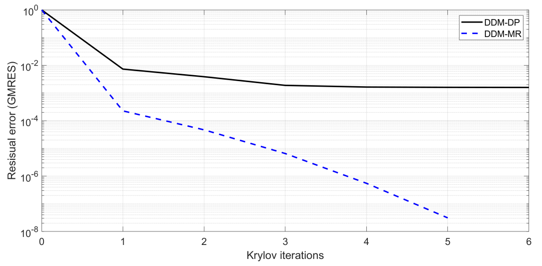


**Figure 4.** Convergence performance of the outer GMRES when solving the EMC problem using the proposed MR and the block diagonal preconditioner in the local solver.

height ($66.7\lambda \times 53.3\lambda \times 13.3\lambda$ at the working wavelength, $\lambda$). The detailed model of this structure includes a integrated communication system in the front of the cockpit as shown in Fig 2. This communication system consist of three sinuous antennas enclosed by a conventional radome (shown at the bottom of Fig 2). The mesh is adapted to the fine detail features of the antennas, providing a total of 1 419 461 unknowns, with a mesh disparity that varies from $\lambda/10$ to $\lambda/350$ in the vicinity of the antennas.

The domain decomposition method was applied together with the multitrace MR preconditioner to obtain the solution of the electric and magnetic current densities on the surfaces of the aircraft and the sensors when the three antennas are excited with a delta-gap at their feeding points. The entire problem was divided into 9 subdomains (shown in the Fig. 2), 6 medium and smooth subdomains belonging to the aircraft fuselage, 2 large subdomains belonging the jet engine inlets (cavities) and 1 medium and complex subdomain corresponding to the dielectric radome and the 3 sinuous antennas. The main complexity of the proposed example is centered on the subdomain containing the antennas. Due to the strong coupling and the high number of unknowns is mandatory the use of a fast solver combined with an efficient preconditioner. Then, the solution is achieved with the use of MLFMA-FFT with the multitrace MR for that complex domain. We use GMRES [19] as a krylov iterative method in the different stages of the DDM.

In order to characterize the performance of the proposed

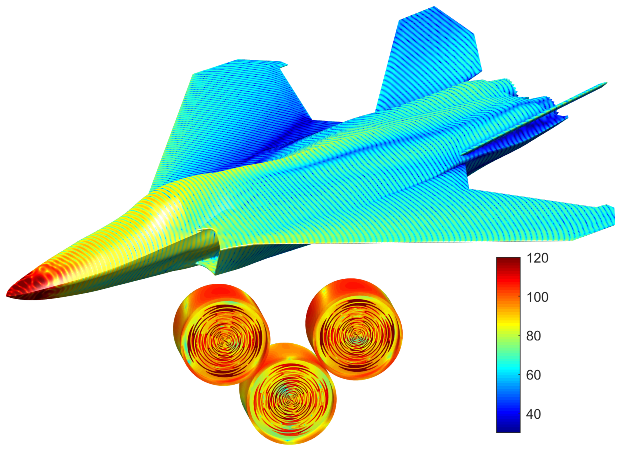


**Figure 5.** Real part of the equivalent electric surface current distribution ($dB\mu A/m$) provided by the SIE-DDM approach. Detailed view of the current density on the antennas surfaces

method, the solution of the domain decomposition method with multitrace MR preconditioner is compared to the solution of the same domain decomposition scheme aplying a block diagonal preconditioner to the radome subdomain. Figure 3 shows the convergence of the iterative solution of the problematic subdomain at the first DDM iteration. At this figure, the convergence of the proposed multitrace MR is compared with the convergence of a diagonal preconditioner applied to the antennas. The use of MR preconditioner strongly reduce the number of iterations required for the solution of the subdomain.

Now, in the Fig. 4, we can observe the effect of applying the two methods used in the Fig. 3 for the solution of the radome domain in the performance of the global DDM solution. Figure 4 shows the convergence of the DDM for the two alternatives, and we can see how let alone to obtain the same behavior, the convergence of the diagonal preconditioner stacks, being impossible the obtention of results, while in the case of using the proposed combination between DDM and multitrace MR, the convergence maintains stable and provides an accurate solution. Finally, the equivalent current density on the aircraft surfaces obtained with the proposed scheme is shown in the Fig. 5, with a detailed view of the currents on the antennas surfaces in the inset.

## 4 Conclusion

In this work, a multitrace multiresolution preconditioner was efficiently combined with the domain decomposition method to speed up the calculation of the local stage in the solution of complex problems including composite multimaterial objects with different levels of multiscale. The efficiency and accuracy of the proposed method was demonstrated through the solution of a numerical example.

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