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Solving differential equations with reconfigurable Mach–Zehnder interferometer photonic networks

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Abstract

In this work we present and discuss the case of a reconfigurable, Mach–Zehnder interferometer-based photonic network, specially designed for implementing any arbitrary matrix operator. After a brief discussion on the available architectures we describe the main elements of the proposed architecture. Finally, using simulations we demonstrate a practical example where an 11×11 MZI network is implement for the solution of differential equation.

1 Introduction

The emerging field of analog computing with photonic devices and metastructures is attracting an increasing research interest, especially for the cases where computationally intensive mathematical operations are performed directly at the hardware level, i.e., utilizing specially designed ultrafast, low-power, wave-based or electronic structures and networks that perform these operations [1, 2, 3]. For instance, such an inverse-designed metastructure was implemented to demonstrate the solution of integral equations with an all-wave approach [4], opening new avenues for the exploration of wave/photonic devices for mathematical processes.

The main building-block for constructing the required reconfigurable photonic networks/operators is a four-port (two inputs, two outputs) tunable coupler, also know as Mach–Zehnder Interferometer (MZI) [5, 6, 7, 8, 9]. This device enables the control over the phase and amplitude of a given input. The proper connection between these MZIs facilitates the implementation of any given operator/kernel.

Here, inspired by the aforementioned works, we expand the introduced metastructure concept of [4] in two directions: (a) by proposing a reconfigurable structure that can implement any given operator and (b) by generalizing the available matrix-inversion capabilities, e.g., towards the solution of differential equations and other linear problems.

2 Solving equations with MZI networks

Perhaps the oldest category for calculating matrix inverses, hence solving linear equations, at the hardware level are networks of optical lenses and filters [10, 11]. These networks have been used for performing different kind of operators, however all in the Fourier optics domain. Key feature is the necessary feedback mechanism that enables the equation solving/matrix-inverting capabilities.

In photonic network technology, there are mainly two ways for implementing a given operator using an assembly of MZIs. These approaches require the mathematical decomposition of the desired operator using (a) a QR decomposition (RZBB/CHMKW architecture) [5, 12, 13], and (b) an SVD decomposition (Miller architecture) [6, 13]. Alternative approaches follow the example of field programming gate arrays but for their photonic counderparts, e.g., utilizing photonic waveguides as in [14]. In every case, the proper operator implementation requires several *a-priori* mathematical calculations, i.e., solution of an eigenvalue/decomposition problem (QR or SVD) or via the solution of a system of equations.

Inspired by the above contributions, we propose a general architecture that can be used for implementing any arbitrary operator in simple and intuitive manner without the need of any *a-priori* mathematical calculations. The architecture can be seen in Fig. (1). The main kernel/operator, i.e., a $N \times N$ matrix, consists of four stages: (a) the ingress, (b) the middle, (c) the egress, and (d) the gain stage. The first stage comprises of a series of N power splitters, each dividing the input signal into N equal components with the same phase as the input phase, but with $1/\sqrt{N}$ of the input magnitude. Each splitter can be realized either as a cascade of $N-1$ MZIs with fixed (θ, ϕ) pairs, or a specially designed photonic splitter, e.g., via inverse design [15]. All the outputs are directed to the middle stage where a dedicated MZI controls the required matrix component values. Next, these signals are properly recombined to the egress stage, where N combiners, i.e., the opposite of the ingress splitters, recombine the output signals. Finally, the gain stage offers the required amplification to the output signals for covering the general cases of any non-unitary operators [8]. The output signal is then directed to the input via a feedback loop, implemented by N directional couplers.

The feedback loop essentially implements a type of Richardson iterative scheme [16]. The proper choice of the

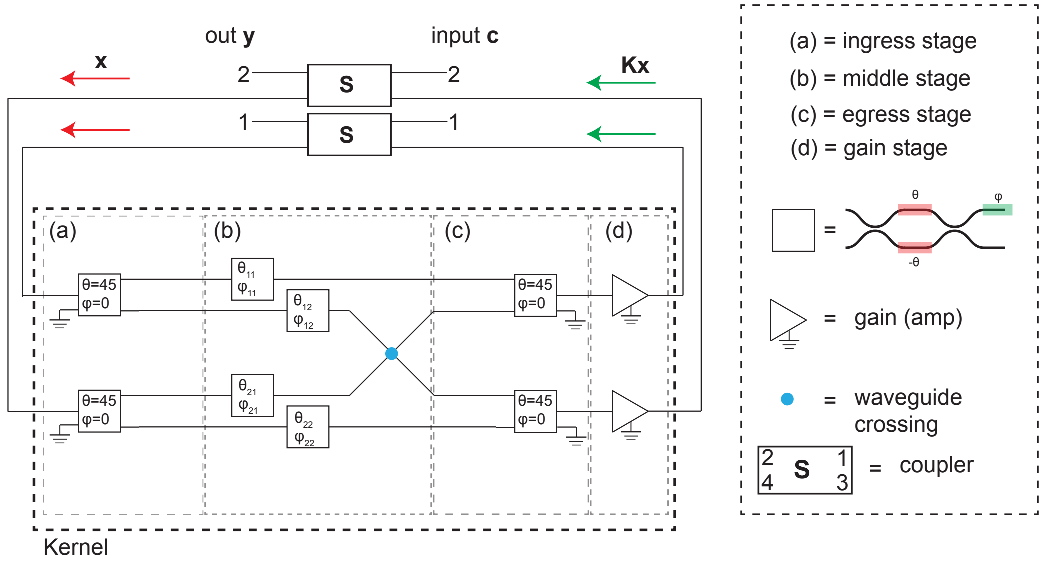

Figure 1. Schematic of the proposed architecture for a 2×2 example operator. The inset pictures provide a legend explaining the basic elements used, i.e., the MZI, the coupler and the amplifier. The open loop kernel has four stages, i.e., (a) ingress, (b) middle, (c) egress, and (d) gain stage. For each stage we have: (a) the ingress stage consists of 2 splitters. Each splitter divides each input signal into 2 identical output signals each with amplitude $\frac{1}{\sqrt{2}}$ relative to the input, hence, a total of 4 outputs are created after the ingress stage. (b) These 4 outputs of are connected with 4 MZIs, each representing a complex-valued element of the 2×2 matrix/operator. The egress stage (c) opposite the ingress stage, i.e., 2 MZIs that re-combine the 4 outputs into to 2 outputs. Finally, the gain stage (d) restores any path losses and amplifies the output for the case of non-unitary matrices.

kernel can give the solution to the general linear problem, $\mathbf{Ax} = \mathbf{b}$. More details regarding the mathematical theory required for this iterative technique will be discussed in the presentation.

3 Discussion

Since the aforementioned network can implement any linear operator, we focus our attention on a more practical case, i.e., solving linear differential equations. For example, we utilize an 11×11 MZI structure for solving a popular differential equation appearing in many quantum mechanics and optics problems, the Airy equation:

$$\frac{d^2 f(x)}{dx^2} + (k)^2 x f(x) = 0 \tag{1}$$

where k can be an arbitrary complex parameter. The general solution of the above equation is described in terms of the Airy functions of the 1st and 2nd order, $\mathrm{Ai}(x), \mathrm{Bi}(x)$, i.e.

$$f(x) = c_1 \mathrm{Ai}\left(-\frac{k^2}{k^{4/3}}x\right) + c_2 \mathrm{Bi}\left(-\frac{k^2}{k^{4/3}}x\right) \tag{2}$$

Finding a solution of the above equation requires the transformation of the continuous problem into a corresponding discrete version via the finite differences (FD) method. The resulting FD problem can be expressed in matrix form $\mathbf{Ax} = \mathbf{b}$, hence the unknown solution can be approximated as the matrix inversion of this linear problem using our MZI system.

Assuming a Dirichlet-type boundary-value problem, i.e., $f(0) = 1, f(1) = 0$, for $k = 2\pi$ we have an analytical solution that can be seen in Fig (2). The retrieved simulation results by the MZI implementation (yellow) indicate an excellent agreement between both the analytical solution (solid blue line) and the numerical solution using a MATLAB code (red dots). Note that the MATLAB code implements a software version of the FD methodology used with the MZI networks. A detailed theoretical analysis and other mathematical subtleties will be further expanded and discussed in the presentation.

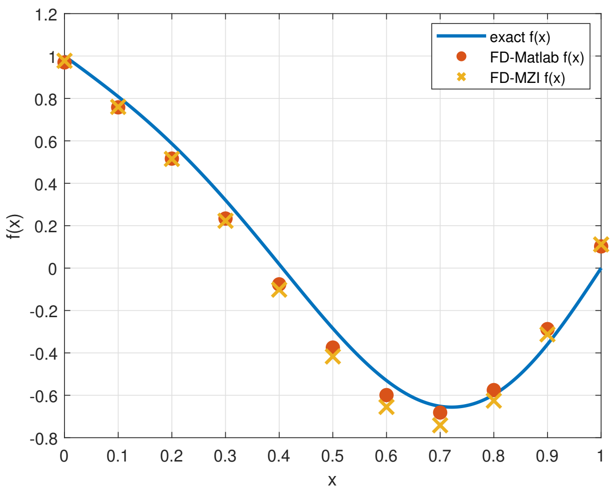

Figure 2. Comparison between the exact solution (solid lines), the computationally inverted matrix (solid red dots) and the retrieved simulation results following the iterative MZI method (orange cross) for a 1D Airy equation for $k = 2\pi$ at the interval $x \in [0, 1]$. The agreement between the analytical and the numerical methods is excellent. More technical details will be discussed in the presentation.

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