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Scattering of an Obliquely Incident Plane Quasi-Electrostatic Wave by a Metal Cylinder in a Magnetoplasma

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Abstract

In this paper, we analyze scattering of an obliquely incident plane quasi-electrostatic wave by a metal cylinder parallel to the ambient magnetic field in a plasma. The electromagnetic field of scattered wave and its directivity pattern are found analytically. Calculations are performed for the typical parameters of the whistler mode frequency range in the Earth's magnetosphere.

1 Introduction

In the near-Earth plasma, it is very common that the quasi-electrostatic (QE) waves are received by spacecraft-borne antennas. We mention here (a) the two-point tethered rocket experiment OEDIPUS-C in which the QE wave packets were transmitted and received by electric dipoles in the ionosphere [1] and (b) detection of QE whistler-mode chorus emissions onboard THEMIS spacecraft in the magnetosphere [2] as the examples of the reception of QE waves of artificial and natural origin, respectively.

One of the problems concerning reception of the QE waves using antennas in magnetoplasmas is the correct calculation of the antenna effective, or electric, length l_{eff}. Indeed, it was shown using the fundamental principle of reciprocity (which is also correct in gyrotropic media) that l_{eff} can be ~ 10 times larger than the antenna geometric length (see [3] and references therein). Such a difference is because of strong reradiation of the incident QE wave by an antenna. However, the reradiation problems can obviously be considered as the scattering, or diffraction, problems. Therefore, it is important to develop another approach to l_{eff} calculation, namely, the approach based on the QE wave scattering problem.

In general, the QE wave scattering problem can be solved only numerically, e.g., using the method of moments, which, as it was shown in [4], works even in the case when a kernel of the corresponding integral equation has a singularity due to the resonance cone. However, it is important to have analytical solutions in some simple cases in order to be able to verify numerical methods. Consequently, in this paper we analyze scattering of an obliquely incident plane QE wave by a metal cylinder parallel to the ambient magnetic field in a plasma. Similar problems have been studied for many years: some preliminary remarks concerning 2D problems of diffraction in anisotropic media are found in [5]; in [6], a 2D problem of diffraction by a conducting circular cylinder clad by an anisotropic plasma sheath was solved; in [7], an infinite cylindrical antenna in a concentric sheath of free space immersed in an anisotropic plasma was analyzed; in [8], the same problem was analyzed but the sheath was neglected; scattering by an infinite cylinder coated with an inhomogeneous and anisotropic plasma sheath was considered in [9]; a very good explanation of the theory of radiation and scattering in anisotropic media is found in [10]. However, it has not been studied yet how a plane QE wave is scattered by a metal cylinder in a magnetoplasma.

2 Formulation of the Problem and Basic Equations

We consider a perfectly conducting infinitely long circular cylinder of radius a parallel to the ambient magnetic field in a plasma (see Figure 1). The electric and magnetic fields of the incident QE wave, with the time factor $\exp(i\omega t)$ dropped, are

$$\vec{E}^{(\text{i})} = \vec{E}_0^{(\text{i})} \exp[-ik_0(qy+pz)], \tag{1}$$

$$\vec{H}^{(\text{i})} = \vec{H}_0^{(\text{i})} \exp[-ik_0(qy+pz)] \tag{2}$$

where $k_0 = \omega/c$ is a wavenumber in free space, $p = (k^{(\text{i})}/k_0)\cos\theta_{\text{res}}$, $q = (k^{(\text{i})}/k_0)\sin\theta_{\text{res}}$, $k^{(\text{i})}$ is a wavenumber of the incident wave, and θ_{res} is the resonance angle. The QE wave fields (1) and (2) satisfy equations

$$\operatorname{rot}\vec{E}^{(\text{i})} = 0, \quad \operatorname{rot}\vec{H}^{(\text{i})} = i\omega\varepsilon_0\hat{\varepsilon}\vec{E}^{(\text{i})}, \tag{3}$$

where ε_0 is the electric constant,

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}$$

is the cold plasma dielectric tensor (so $\cot^2\theta_{\text{res}} = |\varepsilon/\eta|$),

$$\varepsilon = 1 + \frac{\omega_{\text{p}}^2}{\omega_{\text{c}}^2 - \omega^2}, \quad g = -\frac{\omega_{\text{p}}^2\omega_{\text{c}}}{(\omega_{\text{c}}^2 - \omega^2)\,\omega}, \quad \eta = 1 - \frac{\omega_{\text{p}}^2}{\omega^2},$$

ω_c and ω_p are the cyclotron frequency and the plasma frequency of electrons, respectively. Here we neglected the contribution of ions which is possible under the condition $\omega \gg \omega_{LH}$, where ω_{LH} is the lower hybrid frequency. In what follows, we consider the whistler mode frequency range so $\varepsilon > 0$, $\eta < 0$. Constants $\vec{E}_0^{(i)}$ and $\vec{H}_0^{(i)}$ in (1) and (2) are related to each other because of (3). We assume that $|\vec{E}_0^{(i)}| = 1$ in this paper.

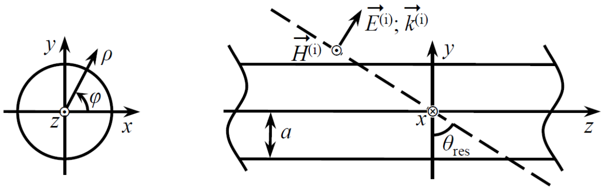

Figure 1. Geometry of the problem.

The electric and magnetic fields of a scattered wave that we are to find satisfy equations

$$\mathrm{rot}\vec{H} = i\omega\varepsilon_0\hat{\varepsilon}\vec{E}, \quad \mathrm{rot}\vec{E} = -i\omega\mu_0\vec{H} \tag{4}$$

where μ_0 is the magnetic constant. In cylindrical coordinates (ρ, φ, z) (see Figure 1), we represent these fields as

$$\begin{bmatrix} \vec{E}(\rho,\varphi,z) \\ \vec{H}(\rho,\varphi,z) \end{bmatrix} = \sum_{m=-\infty}^{+\infty} \begin{bmatrix} \vec{E}_m(\rho) \\ \vec{H}_m(\rho) \end{bmatrix} \exp(-im\varphi - ik_0pz). \tag{5}$$

Substitution of these expansions into (4) gives equations for longitudinal components of the fields in sum (5):

$$\hat{L}_m E_{m,z} - k_0^2 \frac{\eta}{\varepsilon}\left(p^2 - \varepsilon\right) E_{m,z} = -ik_0^2 \frac{g}{\varepsilon} p Z_0 H_{m,z}, \tag{6}$$

$$\hat{L}_m H_{m,z} - k_0^2 \left(p^2 + \frac{g^2}{\varepsilon} - \varepsilon\right) H_{m,z} = ik_0^2 \frac{g}{\varepsilon} \eta p Z_0^{-1} E_{m,z}, \tag{7}$$

where Z_0 is the impedance of free space,

$$\hat{L}_m = \frac{\mathrm{d}^2}{\mathrm{d}\rho^2} + \frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho} - \frac{m^2}{\rho^2}.$$

The solution of these equations [both $E_{m,z}(\rho)$ and $H_{m,z}(\rho)$] is a cylinder function of order m [11]. As for the case considered here we note that the solution should correspond to the outgoing waves of energy (since $\varepsilon > 0$ and $\eta < 0$, the waves of phase propagate in the opposite direction, from $\rho = +\infty$ to $\rho = a$, if we limit ourselves within the transverse, with respect to the anisotropy axis, direction). Consequently, for $\exp(i\omega t)$, the solution is

$$E_{m,z} = \frac{i}{\eta}\sum_{k=1}^{2} A_{k,m} n_k q_k H_m^{(1)}(k_0 q_k \rho), \tag{8}$$

$$H_{m,z} = -\frac{1}{Z_0}\sum_{k=1}^{2} A_{k,m} q_k H_m^{(1)}(k_0 q_k \rho), \tag{9}$$

where $A_{k,m}$ is a coefficient, $H_m^{(1)}(\cdot)$ is the Hankel function of the first kind of order m,

$$n_k = -\frac{\varepsilon}{pg}\left(p^2 + q_k^2 + \frac{g^2}{\varepsilon} - \varepsilon\right),$$

$$q_k^2(p) = \frac{1}{2\varepsilon}\times \left[\varepsilon^2 - g^2 + \varepsilon\eta - (\eta+\varepsilon)p^2 + (-1)^k D^{1/2}\right],$$

$$D = (\eta-\varepsilon)^2 p^4 + 2\left[g^2(\eta+\varepsilon) - \varepsilon(\eta-\varepsilon)^2\right] p^2 + \left(\varepsilon^2 - g^2 - \varepsilon\eta\right)^2,$$

The terms with $k = 1$ correspond to the evanescent O mode (q_1 is purely imaginary), and the terms with $k = 2$ correspond to the propagating X mode ($q_2 > 0$). Though the terms with $k = 1$ vanish at some distance from the cylinder, they contribute to the field structure in its vicinity.

The other components of $\vec{E}_m(\rho)$ and $\vec{H}_m(\rho)$ are represented in terms of $E_{m,z}(\rho)$ and $H_{m,z}(\rho)$. The resulting expressions have the following form [11]:

$$E_{m,\rho} = -\sum_{k=1}^{2} A_{k,m}\left[\frac{n_k p + g}{\varepsilon} H_{m+1}^{(1)}(k_0 q_k \rho) + q_k \frac{n_k p + \eta}{\eta\left(p^2 - P_x^2\right)} \frac{m}{k_0\rho} H_m^{(1)}(k_0 q_k \rho)\right],$$

$$E_{m,\varphi} = i\sum_{k=1}^{2} A_{k,m}\left[H_{m+1}^{(1)}(k_0 q_k \rho) + q_k \frac{n_k p + \eta}{\eta\left(p^2 - P_x^2\right)} \frac{m}{k_0\rho} H_m^{(1)}(k_0 q_k \rho)\right],$$

$$H_{m,\rho} = -\frac{i}{Z_0}\sum_{k=1}^{2} A_{k,m}\left[p H_{m+1}^{(1)}(k_0 q_k \rho) + q_k \frac{p\eta + n_k P_x^2}{\eta\left(p^2 - P_x^2\right)} \frac{m}{k_0\rho} H_m^{(1)}(k_0 q_k \rho)\right],$$

$$H_{m,\varphi} = -\frac{1}{Z_0}\sum_{k=1}^{2} A_{k,m}\left[n_k H_{m+1}^{(1)}(k_0 q_k \rho) + q_k \frac{p\eta + n_k P_x^2}{\eta\left(p^2 - P_x^2\right)} \frac{m}{k_0\rho} H_m^{(1)}(k_0 q_k \rho)\right],$$

where $P_x^2 = \varepsilon - g$.

Coefficients $A_{k,m}$ ($k = 1,2$; $m = 0, \pm 1, \pm 2, \ldots$) are found from the boundary conditions for the total electric field at $\rho = a$:

$$(E_z^{(i)} + E_z)|_{\rho=a} = 0, \quad (E_\varphi^{(i)} + E_\varphi)|_{\rho=a} = 0. \tag{10}$$

For each given m, these conditions lead to a linear system of two equations for $A_{1,m}$ and $A_{2,m}$. The resulting expressions for these coefficients are quite complicated and not presented here.

We also note that the directivity pattern for the scattered wave field (derived from the scattered wave electric field $|\vec{E}|$ at $\rho \to +\infty$) in this problem is expressed as

$$D(\varphi) = \frac{d(\varphi)}{\max\left[d(\varphi)\right]} \tag{11}$$

where

$$d(\varphi) = \left| \sum_{m=-\infty}^{+\infty} A_{2,m} \exp\left(-\frac{i\pi m}{2} - im\varphi \right) \right|. \qquad (12)$$

3 Calculation Results and Discussion

We performed calculations for $\omega \approx 9.4 \cdot 10^3$ s^{-1}, $\omega_\mathrm{p}/\omega \approx 5$, $\omega_\mathrm{c}/\omega \approx 17$, and $k^{(\mathrm{i})}/k_0 = 200$. In Figure 2, the directivity patterns are shown for 3 different values of $\chi \equiv k^{(\mathrm{i})} a \sin\theta_\mathrm{res}$.

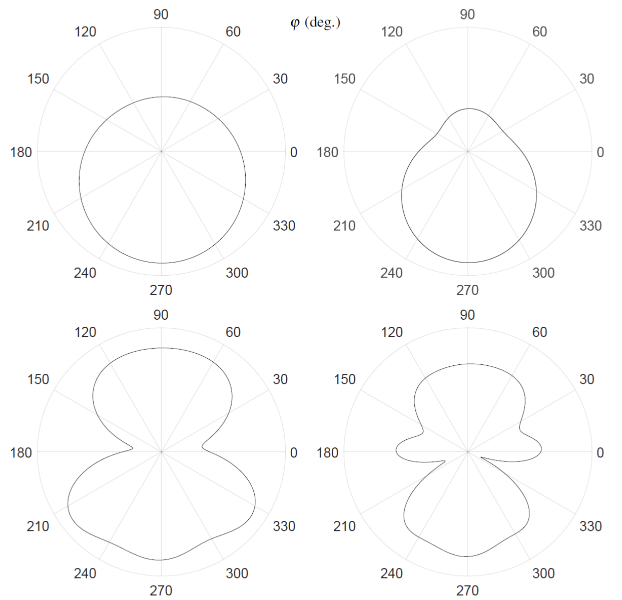

Figure 2. Directivity patterns $D(\varphi)$ for $k^{(\mathrm{i})} a \sin\theta_\mathrm{res} = 1.0$ (top left), 1.5 (top right), 8.0 (bottom left), and 16 (bottom right).

As it follows from Figure 2, when $\chi = 1$, $D(\varphi)$ has one lobe (at $\varphi = 270°$) corresponding to the forward scattering (energy of the incident wave propagates in the direction of $-y$, i.e., from $\varphi = 90°$; see Figure 1). As χ increases, the lobe corresponding to backward scattering (at $\varphi = 90°$) appears and becomes significant. When χ reaches ~ 16, two weak side lobes at $\varphi = 0°, 180°$ appear. Furthermore, it can be shown that more weak side lobes appear as χ becomes larger (e.g., $\chi \sim 100$). The conclusion from Figure 2 is that the directivity pattern becomes more complicated as radius a increases.

Finally we note that the QE waves typically have a continuous and wide spectrum of wave numbers corresponding to the plane waves that propagate along the resonance cone direction. In such cases, the resulting expressions should include an integral over these harmonics. However, in many cases it is appropriate to analyze one harmonic of QE waves only. This may occur, e.g., when refraction in the magnetosphere is significant: in this case, a very narrow wave packet is detected at the observation point because initially wide packet spreads due to the fact that different harmonics have different ray trajectories.

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