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Spacetime Metamaterials: from Synthetic Fresnel Drag to Nonreciprocal Amplification

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Abstract

The advent of tunable materials has sparked renewed interest in metamaterials which combine spatial and temporal degrees of freedom, opening new avenues for advanced functionalities beyond the conventional limitations of static metamaterials, such as reciprocity and energy conservation. In this contribution we show that a simple traveling-wave modulation in the electromagnetic parameters of a medium yields completely new regimes of light propagation as the phase velocity of the modulation approaches, and surpasses, the speed of light. We unveil, analytically and numerically, three novel regimes of light-matter interaction which can stem from a spatiotemporal modulation in the electromagnetic parameters of a medium: (1) synthetic Fresnel drag arising in non-moving media, (2) broadband, dynamical localization of light and (3) a new broadband nonreciprocal amplification mechanism. Remarkably, the temporal frequency of the modulation can, in principle, be very slow compared to the frequency of the wave, the ultimate limitation being only the lifetime of the electromagnetic excitation itself, generalizing the conventional parametric amplification concept.

1 Introduction

Temporal control of light is a long-sought dream of electromagnetics, with some of the key ideas already introduced by Cullen [1], Cassedy and Oliner [2], More recently the current quest for magnet-free electromagnetic nonreciprocity and isolation [3,4] has driven immense efforts in the breaking of reciprocity via spatiotemporal modulation of the parameters ε and μ of a medium [5].

In this summary paper we explore three different regimes of spatiotemporal modulation, all of which are capable of breaking reciprocity throughout phase space, and which can drive completely different behavior in the waves propagating through a medium. We assume that the dielectric permittivity (or, similarly, the magnetic permeability μ) of a medium is modulated in a traveling-wave fashion $\varepsilon(x,t)=\varepsilon_0\varepsilon_1[1+2\alpha_\varepsilon\cos(gx-\Omega t)]$. As illustrated in Fig. 1, the conventional band structure defined for time-independent crystals (panel a) can be generalized to that for a spatiotemporal one by defining oblique space-time reciprocal lattice vectors $\mathbf{\Gamma}=(g,\Omega)$.

The angle that the vector $\mathbf{\Gamma}$ makes with the k-axis thus defines the velocity of the modulation grating, which is

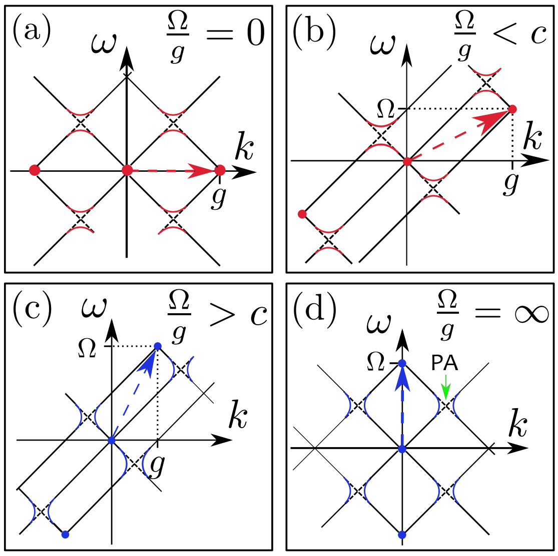

Figure 1. Band structure for gratings with spatial (a), spatiotemporal (b: subluminal, c: superluminal), and temporal (d) modulation.

given by $c_g=\Omega/g$. For subluminal grating velocities $c_g<c_0$ (panels a and b) conventional band gaps open, characterized by imaginary eigenfrequencies, whereas $c_g>c_0$ lead to unstable, so-called k-gaps (panels c and d). Finally, as $c_g=\Omega/g\rightarrow\infty$, the system becomes a purely time-modulated one, whose parametric amplification capabilities are well-known (panel d).

As already shown by Cassedy and Oliner [2], the solutions to Maxwell's equations for these systems converge to well-defined Floquet-Bloch waves only outside of a so-called "luminal" velocity regime:

$$\frac{1}{\sqrt{1+2\alpha_\epsilon}\sqrt{1+2\alpha_\epsilon}}\leq c_g/c_0\leq\frac{1}{\sqrt{1-2\alpha_\epsilon}\sqrt{1-2\alpha_\epsilon}},\qquad(1)$$

where $c_0=1/\sqrt{\varepsilon_0\varepsilon_1\mu_0\mu_1}$ is the wave velocity in the unmodulated medium, and we shall refer to the lower and upper velocity bounds as c^- and c^+ respectively. Based on this fact, we can distinguish three regimes of operation, based solely on the phase velocity c_g of the grating:

1. Off-luminal ($c_g<<c^-$ and $c_g>>c^+$)
2. Quasi-luminal ($c_g\lesssim c^-$ and $c_g\gtrsim c^+$)

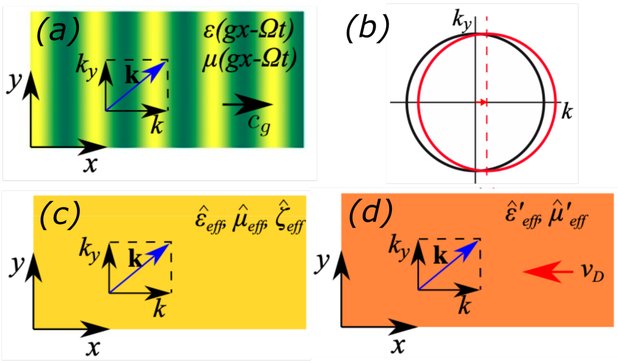

Figure 2. A spatiotemporal modulation in the dielectric and magnetic parameters of a medium is capable of inducing synthetic Fresnel drag, shifting the isofrequency contours (b). The spacetime metamaterial can be equivalently represented as an effective magnetoelectric (i.e. bi-isotropic) medium © or as a moving uniaxial one (d).

3. Luminal ($c^- < c_g < c^+$),

which can be attained by independently varying the temporal and spatial frequencies (Ω and g) of the traveling-wave modulation.

2 Off-luminal regime: synthetic Fresnel drag

This regime, whereby the phase velocity of the modulation is much lower (or much higher) than the waves in the unmodulated medium, is by far the most widely investigated to date, partly due to the technological challenges involved in realizing fast modulation speeds, or very long spatial modulation periods $L = 2\pi/g$ required for $c_g \to c_0$. The typical purpose of these systems is the realization of isolation, i.e. unidirectional propagation of waves through the system, by exploiting the asymmetric band-gap (see Fig. 1b-c) which ensures that either forward or backward waves are reflected when impinging on the medium.

Less attention has been devoted to the behavior of this system in the long wavelength limit, which is the regime where the grating effectively behaves as a metamaterial. In this limit, the dispersion relation appears exactly symmetric about the frequency axis, i.e. the system is reciprocal in the long-wavelength (quasi-static) limit. This observation has recently led to the conclusion that the Fresnel drag, by which light propagating in a moving medium speeds up or slows down as dictated by special relativity, does not exist for spatiotemporally modulated systems. However, this statement no longer holds true when both the dielectric permittivity ε and the magnetic permeability μ are modulated [6].

By Fourier expanding the Floquet-Bloch solutions to Maxwell's Equations for these media, assuming s-polarization, and choosing a basis of forward and backward traveling waves, we derive the following eigenvalue problem for the in-plane wavevector k:

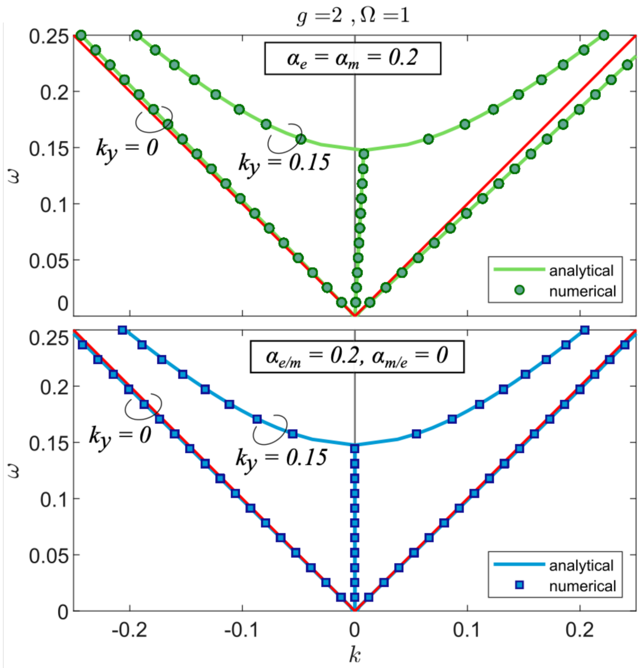

Figure 3. Nonreciprocity at long wavelengths, corresponding to a Fresnel drag, occurs in spacetime media if both the dielectric and the magnetic parameter of the material are modulated (top panel). If any of the two modulation is switched off, the band structure reverts to a reciprocal one (bottom panel).

$$\begin{pmatrix} \mathbf{M}^{++} & \mathbf{M}^{+-} \\ -\mathbf{M}^{+-} & \mathbf{M}^{--} \end{pmatrix} \begin{pmatrix} \mathbf{E}+\mathbf{H} \\ \mathbf{E}+\mathbf{H} \end{pmatrix} = k \begin{pmatrix} \mathbf{E}+\mathbf{H} \\ \mathbf{E}-\mathbf{H} \end{pmatrix} \tag{2}$$

where $\mathbf{E}$ and $\mathbf{H}$ are vectors containing the Fourier amplitudes of the z and y components of the electric and magnetic field, respectively, while $\{\mathbf{M}^{\mathbf{ij}}\}$ are matrices which contain the coupling between Fourier components.

Diagonalizing the matrix on the left hand side by including coupling between 3 modes, and taking the long-wavelength limit $k \to 0$ yields the dispersion relation:

$$\beta^2\omega^2 = \kappa^2 k_y^2 + (k - \delta\omega)^2 \tag{3}$$

where:

$$\beta^2 = c_g^{-2}\left(1 + \frac{2\alpha_\varepsilon^2}{(c_0/c_g)^2 - 1}\right)\left(1 + \frac{2\alpha_\mu^2}{(c_0/c_g)^2 - 1}\right) \tag{4}$$

$$\kappa^2 = 1 + \frac{2\alpha_\mu^2}{(c_0/c_g)^2 - 1} \tag{5}$$

$$\delta^2 = \frac{2\alpha_\varepsilon\alpha_\mu}{c_0^2/c_g - c_g} \tag{6}$$

Thus, the dispersion contours in the $k - k_y$ plane consist of ellipses with average radius $\beta\omega$. We note, however, that the center of these iso-frequency contours is displaced from the origin along the k-axis by $\delta\omega$. This

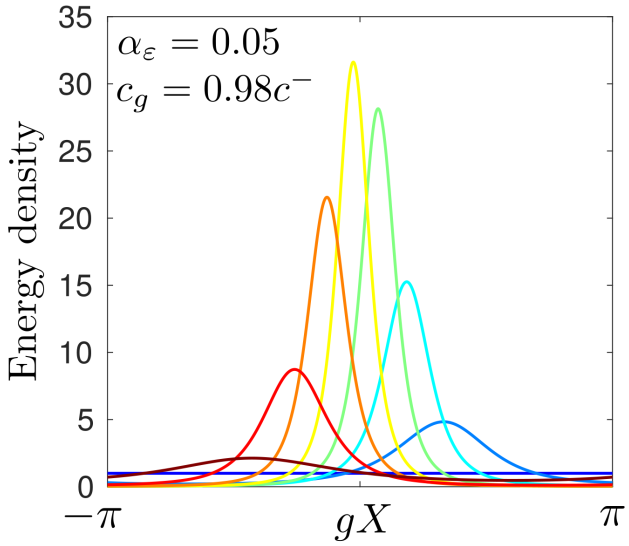

Figure 4. Dynamical localization of light in a quasi-luminal spacetime medium. As time passes (blue to red curves), any incident monochromatic plane wave is compressed and decompressed as a result of the spatiotemporal modulation.

asymmetry, which vanishes with the effective parameter δ when either the dielectric modulation amplitude α_ε or its magnetic counterpart α_μ are null, is a signature of nonreciprocity: remarkably, reciprocity in these media is broken in the long-wavelength limit, which is physically equivalent to a drag, analogous, although not quantitatively equivalent, to that observed by Fizeau in 1851, but existent despite matter being physically at rest.

In fact, this system can be mapped to an effective static medium with magnetoelectric coupling (i.e. bi-isotropic), the latter contribution vanishing when either the dielectric or the magnetic modulations are switched off, as well as to a moving medium with uniaxial anisotropy [6].

4 Quasi-luminal regime: periodic localization

In this regime the grating velocity c_g approaches the instability threshold c^- (from below) or c^+ (from above), so that the modulation amplitudes of the electromagnetic parameters $2\alpha_\varepsilon$ and $2\alpha_\mu$ can make the local phase velocity $c_p(x,t) = 1/\sqrt{\varepsilon(x,t)\mu(x,t)}$ of the waves periodically approach the velocity of the grating.

In the grating frame, this implies that the waves slow down, compressing into a narrow pulse, as they approach the point where the difference $|c_g - c_p(x,t)|$ between the grating speed and their phase velocity reaches a minimum (Fig. 4). As time passes, however, the pulse can tunnel through the compression point to the opposite side of the modulation profile, it is sharply decompressed, its energy density redistributing itself evenly across the grating period, so that the wave returns, once again to its original monochromatic state.

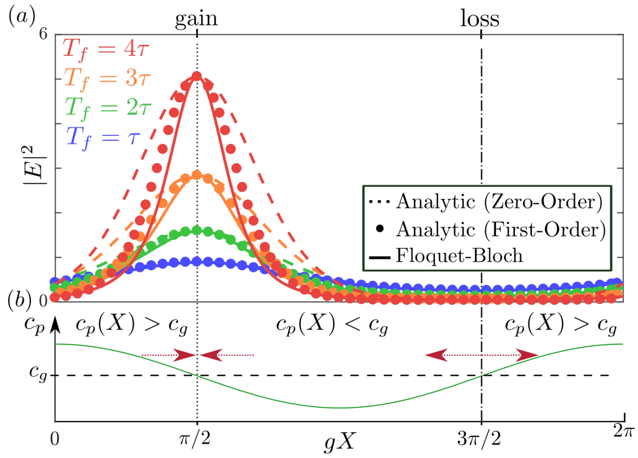

Figure 5. Broadband nonreciprocal amplification in a luminal medium results from light-trapping at the points where the velocity of the grating matches the local phase velocity of the waves. This mechanism is capable of generating harmonics at an exponential rate, starting from any input frequency, even DC.

In the superluminal scenario, the waves can thus escape a grating period by moving faster than the modulation phase, whereas in the sub-luminal case they can fall behind, and this cycle repeats periodically.

Remarkably, these oscillations occur, identically, for any incident frequency of the wave, and therefore do not result from a conventional beating between the two frequencies in the system: the dynamical, periodic localization of the waves as they propagate through a quasi-luminal medium is a broadband phenomenon [7].

3 Luminal regime: broadband amplification

As the grating velocity surpasses the threshold of the unstable regime, the modulation of the local velocity is able to realize points $(\tilde{x}, \tilde{t})$, within each grating period, where the grating velocity c_g matches the local phase velocity

$$c_p(\tilde{x}, \tilde{t}) = \frac{1}{\sqrt{\varepsilon(\tilde{x}, \tilde{t})\mu(\tilde{x}, \tilde{t})}} \tag{7}$$

of the wave [8].

In this regime, electromagnetic waves of any frequency are trapped within a grating period, unable to escape. By exponentially compressing the lines of force at the points $(\tilde{x}, \tilde{t})$, the medium amplifies any wave, including a static electric field (or magnetic, in the case of a modulation of the magnetic permeability) into arbitrarily intense and short pulses, as shown in Fig. 5.

By visualizing the system in the reference frame of the grating, where its profile varies periodically along the comoving coordinate $X = x - c_g t$, it becomes clear that the compression of the waves arises because their local velocity on the left of the accumulation point $gX = \pi/2$

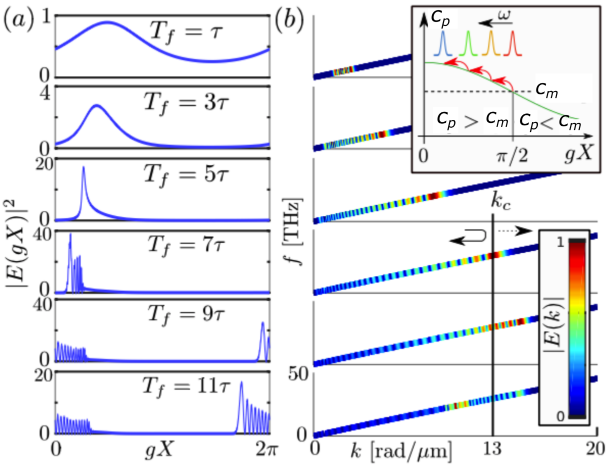

Figure 6. Amplification in a luminal medium breaks down as a result of finite group velocity dispersion. The higher frequencies of the pulse propagate at a lower velocity. Hence they localize at a point where the velocity modulation compensates for their lower phase velocity. Finally, the pulse breaks as higher frequency components are too slow to be trapped, thus falling back into the quasi-luminal regime.

is higher than that of the grating, whereas they cannot keep up with it in region ahead, where $c_p < c_g$. Note that another point exists, where the two velocities match. However, the opposite mechanism occurs there: the waves are decompressed, reducing their energy, which is absorbed by the modulation mechanism. Finally, the system is completely transparent to backward waves, representing a highly desirable feature for amplification devices, as it would protect the light sources by avoiding the amplification of any spurious reflections.

As opposed to conventional parametric amplifiers, where amplification requires that a modulation mechanism is able to couple positive and negative frequencies, the only limiting factor for amplification in these systems is given by the lifetime of the excitation, which needs to be lower than the temporal frequency of the modulation, which is responsible for the amplification, in order for amplification to prevail over loss. This amplification mechanism, new to the best knowledge of the authors, contrast with the conventional amplification mechanisms, e.g. lasers, which rely on the addition of electromagnetic field lines into the system. In fact, the amplification mechanism which governs spacetime metamaterials does not arise from any imaginary eigenvalues, but from the unstable excitation of higher harmonics [9].

By using two-dimensional, tunable materials, this unidirectional amplification principle could be exploited to amplify surface waves which obey a linear dispersion relation. One such excitation is the acoustic branch of surface plasmons in double-layer graphene, whose dispersion is approximately linear when the interlayer gap is much narrower than the plasmon wavelength. The results for such a system are shown in Fig. 6 [8]. As the luminal modulation couples the pulse to higher and higher frequency-wavevector harmonics, however, group velocity dispersion becomes significant, determining the ultimate limit of this amplification process. As higher frequencies sample the nonlinearity of the dispersion band, some of these wave components will propagate more slowly than the rest of the pulse, leading to the reshaping of the pulse.

At a first stage, the slowing down of the high frequency components of the pulse skews the latter, as shown in Fig. 6. Ultimately, as some of these frequency components propagate slower and slower, their phase velocity becomes too slow for them to be amplified, and they effectively fall back into the quasi-luminal regime, so that the pulse breaks into a train of narrow peaks [8].

6 Conclusions

In this summary paper, we have provided an overview of some of the effects which spatiotemporal modulation of the electromagnetic parameters of a medium can yield. Interestingly, the access of broadband functionalities such as nonreciprocity and amplification appears as a common trend in many of these exotic systems.

7 References

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