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Spiral Resonators Array in inductive Wireless Power Transfer Applications: an Equivalent Lumped Circuit Retrieval Method

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Abstract

In this paper we introduce a retrieval method to derive an equivalent lumped circuital model for spiral resonators arrays in inductive Wireless Power Transfer applications. We first introduce the theoretical concepts at the basis of the array description through an equivalent *RLC* circuit. Then, we develop a numerical test-case to evaluate the performance of the proposed method, constituted by resonant driving and receiving coils with an interposed spiral resonators array; finally, we compare full-wave simulations and analytical estimations. The observed excellent agreement between full-wave and analytically-retrieved impedance parameters demonstrates the efficiency of our procedure, encouraging further analysis.

1 Introduction

Inductive Wireless Power Transfer (WPT) is a consolidated topic in electromagnetic research that is nowadays receiving a great attention due to its increasingly number of applications [1]. A WPT system is ubiquitously present in a number of different devices: automotive, biomedical implants and commercial devices for wireless recharge are just a few examples of its widespread diffusion [2]-[4].

The considerable WPT scientific and industrial implications have pushed the research to develop strategies to improve the performance (efficiency and working distance) of inductive links. Among the various solutions appeared in the literature, metamaterials and metasurfaces have proved as one of the most promising [5]. As well known, metamaterials are engineered structures able to show non-conventional electromagnetic properties (permittivity and permeability), not found in nature. This surprising behavior is made possible by the metamaterials elementary unit-cells, that are resonant elements characterized by a strongly subwavelength size [6].

In inductive WPT, the most common structure for a metamaterial (or a metasurface if sufficiently thin) consists of an array of resonant magnetic inclusions, like spiral resonators. Spiral resonators arrays are commonly used in inductive WPT to enhance the efficiency and the working distance [5], [7] or to achieve shielding properties (useful for safety purposes) [8]. In a typical arrangement, an opportune resonators array is interposed between the classical 2-coil WPT system (driver/receiver couple).

It is well known that the design and the position of the interposed slab are important design aspects to fully exploit the metamaterial properties [9]. However, in practical cases, the common approach consists in full-wave simulations for the performance evaluation and for the optimization of the slab distance and positioning. This is extremely time-consuming and, in general, brings to a lack about quantification and understanding of the interactions between the slab and the WPT coils.

In order to overcome this limit, we herein introduce an analytical procedure to obtain an equivalent lumped model of the entire metamaterial slab. Our aim is extracting the equivalent *RLC* circuit of the whole resonant structure and its mutual coupling coefficients with the WPT coils. The availability of an equivalent model can be significantly helpful to facilitate the design and to avoid computationally expensive simulations for the optimization of the system.

The paper is organized as follows. In Section 2 we introduce the basic concepts behind the circuital equivalent model approximation; Section 3 is devoted to present a numerical test-case and a performance comparison between the retrieved lumped equivalent and full-wave simulations; finally, Conclusions follow.

2 Methods

In a typical arrangement, an inductive WPT system is constituted by a driving and a receiving coil, both resonant at the same working frequency. To enhance the performance of this simple 2-coil set-up, a metamaterial (i.e., an array of resonant unit-cells) can be interposed.

A very effective way to describe this system is using the network model. Indeed, if we indicate the driver coil with index 1, the N metamaterial array elements with indexes $2\div(N+1)$, and the receiver coil with $(N+2)$, the complete system matrix can be written as follows.

$$\begin{pmatrix} Z_{11} & Z_{12} & \cdots & Z_{1(N+1)} & Z_{1(N+2)} \\ Z_{21} & Z_{22} & \cdots & Z_{2(N+1)} & Z_{2(N+2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{(N+1)1} & Z_{(N+1)2} & \cdots & Z_{(N+1)(N+1)} & Z_{(N+1)(N+2)} \\ Z_{(N+2)1} & Z_{(N+2)2} & \cdots & Z_{(N+2)(N+1)} & Z_{(N+2)(N+2)} \end{pmatrix} \begin{pmatrix} I_1 \\ c_2 I_x \\ \vdots \\ c_{(N+1)} I_x \\ I_{(N+2)} \end{pmatrix} = \begin{pmatrix} V_1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

where the currents flowing in each of the array elements are represented by the equivalent current I_x weighed by the respective current coefficient c_i. This impedance matrix can be easily obtained from a single full-wave simulation.
It is possible to rearrange the equations going from row 2 to (N+1), those relative to the array elements. In particular, summing up the equations and reorganizing the terms, system (1) can be condensed into the equivalent 3-coil model.

$$\begin{pmatrix} Z_{11} & Z_{1x} & Z_{1(N+2)} \\ Z_{x1} & Z_{xx} & Z_{x(N+2)} \\ Z_{(N+2)1} & Z_{(N+2)x} & Z_{(N+2)(N+2)} \end{pmatrix} \begin{pmatrix} I_1 \\ I_x \\ I_{(N+2)} \end{pmatrix} = \begin{pmatrix} V_1 \\ 0 \\ 0 \end{pmatrix} \quad (2)$$

In (2), terms Z_{x1} and $Z_{x(N+2)}$ represent the mutual coupling impedances between slab/driver and slab/receiver; on the other hand, the term Z_{xx} describes the equivalent *RLC* of the adopted metamaterial and it can be expressed as:

$$Z_{xx} = \left(R_x + j\omega L_x + 1/j\omega C_x\right) \quad (3)$$

In this way, the entire slab has been reduced to a simple *RLC* circuit whose interactions with the driver and receiver coils are easier to model, thus avoiding further computationally expensive full-wave simulations.
Specifically, the optimum *RLC* combination can be determined by finding the values of the model that lead to the best fitting with full-wave results.

3 Numerical Test-case

3.1 Adopted WPT Model

In order to verify our proposed retrieval method, we designed a numerical test-case, exploiting an electromagnetic solver based on the Method of Moments (Feko Suite, Altair, Troy, MI, USA).

For simplicity, both the driver and the receiver were identical; they consisted in a 18 cm diameter solenoid with 5 turns, realized with a 4 mm radius lossy copper wire. Both the coils were made resonant at the same working frequency (6 MHz), that was chosen within the typical range for inductive WPT applications. Then, we interposed between the two coils a metamaterial slab made by a 5×5 array of resonant spiral resonators. Each unit-cell consists of a 8-turns planar spiral with a 4 cm diameter, realized with a 1.4 mm diameter lossy copper

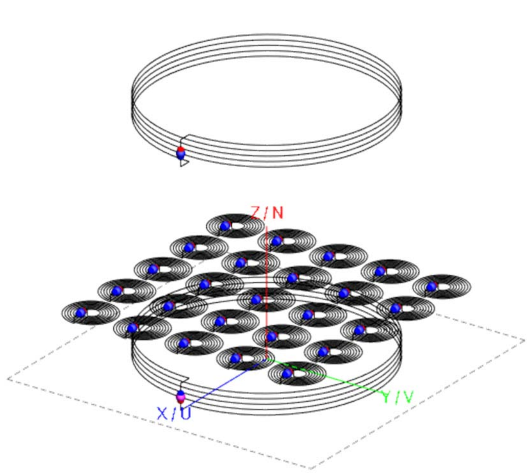


Figure 1. CAD model of the proposed test-case: the spiral resonators array is placed 1 cm away from the driver coil.

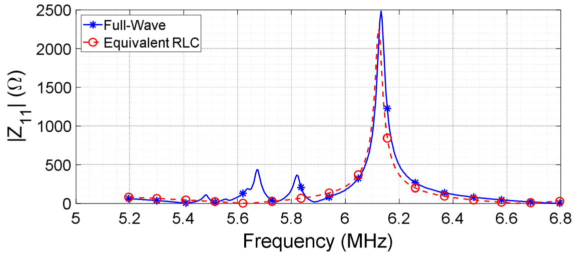
wire. We loaded with an opportune capacitor each cell in order to obtain the entire slab response at 6 MHz. Driver and receiver coil were separated by 16 cm, while the slab is positioned 1 cm above the driver. The CAD model developed in Feko is reported in Fig. 1.

3.2 Results

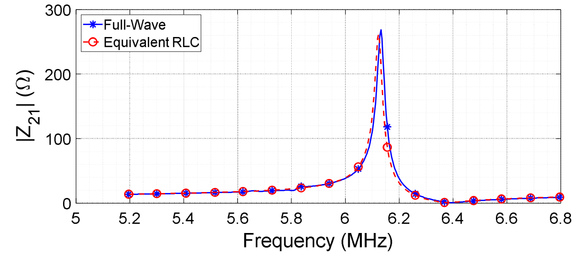
We performed a single full-wave simulation with the system reported in Fig. 1; in this way, we obtained the impedance matrix reported in (1), and therefore extracted the lumped model of the slab.
In addition, from the same simulation, we obtained also the impedance parameters of the 2-port system, i.e. the matrix when the ports were present only at the driver and at the receiver.
Finally, we compared the 2-port impedance matrix obtained from the full-wave simulation and the one estimated through the proposed retrieval method; we found the *RLC* parameters that led to the best fit with the full-wave simulation. The results are reported in Fig. 2. We obtained an excellent agreement between simulation and lumped model, demonstrating the efficacy of the equivalent system. This result is significantly important because, once the *RLC* equivalent is extracted and validated, it is possible to predict the interactions of the slab with the driving and receiving coils for different positions. Thus, the performance optimization and the design procedure can be highly facilitated.

4 Conclusions

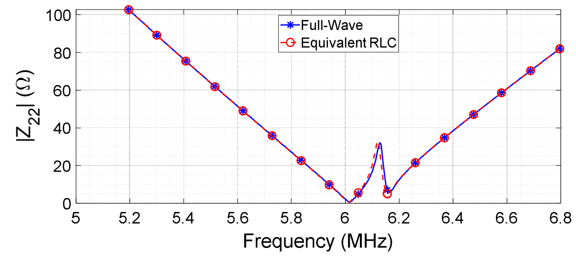
In this paper we introduced a retrieval method to describe an array of resonant elements (i.e. a metamaterial) with its equivalent lumped model for inductive Wireless Power Transfer applications. The extraction method is based on a single full-wave simulation and on an analytical approach able to describe the entire slab through an equivalent *RLC* circuit. We compared the obtained results against full-



(a)



(b)



(c)

Figure 2. Absolute values of the impedance parameters comparison: the lumped model retrieved parameters are in excellent agreement with full-wave simulations.

wave simulations, achieving a very good agreement. This demonstrates that the retrieved *RLC* model is effective in the description of the slab and, thus, it can be considered as its real equivalent.

The important advantage of having an equivalent model of a complex structure relies in the possibility of an easier prediction of the interactions with the other elements of the WPT arrangement, thus facilitating the final design and avoiding computationally expensive full-wave simulations.

Further development can be directed to refine the analytical retrieval method and to apply it for fabricated WPT prototypes.

5 References

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