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Effective Length of a Receiving Antenna in Case of Spacecraft Observations of Quasi-Electrostatic Chorus Emissions

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Abstract

We propose a method of calculating the effective length of receiving antenna for the case of spacecraft observations of quasi-electrostatic chorus emissions. Using the obtained analytical expression, we calculate this length for some measurements of chorus wave quasi-electrostatic fields onboard THEMIS spacecraft. The calculation results show that the effective length can be up to an order of magnitude greater than the geometric length of receiving antenna. Therefore, the actual electric field value can be much less as compared to the one calculated using the geometric length which is a conventionally used technique in the satellite data analysis. In particular, this result can be important for the estimates of electron energization by quasi-electrostatic chorus waves.

1 Introduction

One of the factors that must be considered for the wave electric field measurement in plasmas using antennas is the possible significant change of the antenna effective length. Such a change is known to be most important for the waves propagating in the quasi-electrostatic mode along the so-called resonance cone, e.g., for the whistler-mode waves [1, 2]. For these waves, the problem of calculating the complex amplitude E of the electric field from the complex amplitude U of voltage induced on the receiving antenna is nontrivial. We introduce the effective (or electrical) length l_{eff} of a receiving dipole antenna according to formula

$$U = E l_{\mathrm{eff}} \cos\Upsilon. \tag{1}$$

Factor l_{eff} describes two characteristics of the receiving antenna. Firstly, it implies the dependence of the excitation coefficient of antenna current on angle Υ between the electric field vector and the antenna. Secondly, l_{eff} describes the reradiation efficiency of the antenna. In particular, if the antenna geometric length l_{rec} is much smaller than the electromagnetic wave length λ in a plasma (along the ambient magnetic field), then usually such an antenna effectively reradiates the received quasi-electrostatic waves. In this case $l_{\mathrm{eff}} > l_{\mathrm{rec}}$ [1]. The electric field value, obtained from the voltage data, is therefore not equal to $U/(l_{\mathrm{rec}} \cos\Upsilon)$ in general.

Some general relationships of the receiving antenna theory for magnetized plasmas were developed in the earlier works [1, 2] and were applied to the case when the incident electric field was fully defined by the transmitting dipole antenna with well known geometry.

A different situation takes place for emissions of natural origin. One of the important types of such emissions which can propagate in the quasi-electrostatic whistler mode is the very low frequency chorus in the Earth's magnetosphere, i.e., a series of signals of short duration (about 0.1 to 0.5 s) with rapidly changing (more typically, rising) frequency which is below the local electron gyrofrequency. The chorus waves in the magnetosphere have been observed since 1960s [3]. It is now generally accepted that they are excited due to the electron cyclotron instability in the Van Allen radiation belts [4]. Typically, chorus emissions propagate quasi-parallel to the ambient magnetic field in their source region [5]. However, recent analysis of satellite data shows that chorus can also propagate in the quasi-electrostatic mode close to the resonance cone [6] in the frequency range $\Omega_{\mathrm{LH}} < \omega_0 < \min(\omega_{\mathrm{pe}}, \omega_{\mathrm{ce}})$, where Ω_{LH} is the lower hybrid frequency, $\omega_0 = 2\pi f_0$, f_0 is the radiation frequency at certain time moment, and ω_{pe} and ω_{ce} are the electron plasma and cyclotron frequencies, respectively. The angle between the geomagnetic field and the resonance cone in the space of wave vectors $\vec{k}$ equals $\theta_{\mathrm{res}} = \arctan\sqrt{|\eta_0/\varepsilon_0|}$, where $\varepsilon_0 = \varepsilon(\omega_0)$, $\eta_0 = \eta(\omega_0)$, and $\varepsilon(\omega)$ and $\eta(\omega)$ are the transverse and longitudinal (with respect to the geomagnetic field) dielectric tensor components of cold plasma, respectively.

In the case of chorus waves, the source region characteristics are known only in general in each particular case, and the incident electric field structure, required for the analysis, is not given *a priori*. Therefore, in order to calculate the antenna effective length, we need a model of the effective source which provides the wave field with measured parameters. In this paper, we propose such a model assuming that the wave field is a quasi-electrostatic whistler-mode packet with a spread in wave vectors and develop a method for calculating the effective length of an antenna receiving such a wave packet. Then we apply the obtained formula to spacecraft observations of oblique chorus waves.

2 Receiving Antenna Theory for Spacecraft Observations of Quasi-Electrostatic Chorus Wave

The effective length calculation is based on the reciprocity theorem [1]

$$\int\limits_{\text{pl}} \rho(\vec{r},t)\Phi_0(\vec{r},t)\mathrm{d}\vec{r} = \int\limits_{\text{ant}} \rho_0(\vec{r},t)\Phi(\vec{r},t)\mathrm{d}\vec{r}, \qquad (2)$$

where the integrals are over the plasma ("pl") and antenna ("ant") volumes, $\Phi(\vec{r},t)$ is the scalar potential of the incident wave, $\rho(\vec{r},t)$ is the charge fluctuation in the plasma which induces voltage $U(t)$ on the antenna terminals, and $\rho_0(\vec{r},t)$ and $\Phi_0(\vec{r},t)$ are the antenna charge distribution and its potential (a trial field), respectively. It is easy to find from (2) the spectrum of the induced voltage

$$U_\omega(\omega) = \frac{i}{16\pi^2\sqrt{\varepsilon_0(\varepsilon_0+|\eta_0|)}} \times \int\limits_0^{+\infty}\int\limits_0^{2\pi} \left[\vec{k}\vec{E}_{\vec{k}\omega}(-\vec{k},\omega)\rho_{0\vec{k}}(\vec{k})\right]\Big|_{\theta=\theta_{\text{res}}} \mathrm{d}k\mathrm{d}\psi, \qquad (3)$$

where $k=|\vec{k}|$, ψ is the azimuth angle in $\vec{k}$-space (in the plane transversal to the geomagnetic field $\vec{H}_0$), θ is the angle between $\vec{H}_0$ and $\vec{k}$, $\vec{E}_{\vec{k}\omega}$ is the spatio-temporal spectrum of the incident wave electric field, and $\rho_{0\vec{k}}$ is the spatial spectrum of the charge distribution along the receiving antenna, calculated from $\rho_0(\vec{r},t)$ for a certain frequency ω. If the receiving dipole consists of 2 thin straight rods with a gap between them, then it is appropriate to choose the piecewise constant charge distribution along the wire [7]:

$$\rho_{0\vec{k}}(\vec{k})\Big|_{\theta=\theta_{\text{res}}} = -\frac{4i\exp[-ikR_0(\psi)]}{\gamma(\psi)kL_{\text{rec}}}\sin^2\left[\frac{\gamma(\psi)kL_{\text{rec}}}{2}\right]. \qquad (4)$$

If the receiver consists of 2 small, as compared to the distance between them, spherical conductors placed on the thin rod, then it may be represented as 2 point charges:

$$\rho_{0\vec{k}}(\vec{k})\Big|_{\theta=\theta_{\text{res}}} = -2i\exp[-ikR_0(\psi)]\sin[\gamma(\psi)kL_{\text{rec}}]. \qquad (5)$$

Here we assume without loss of generality that the total half-dipole charge amplitude equals 1; $L_{\text{rec}} = l_{\text{rec}}/2$; $\gamma(\psi) = \sin\alpha\sin\theta_{\text{res}}\cos(\psi-\beta)+\cos\alpha\cos\theta_{\text{res}}$, $R_0(\psi) = x_0\sin\theta_{\text{res}}\cos\psi + y_0\sin\theta_{\text{res}}\sin\psi + z_0\cos\theta_{\text{res}}$; x_0, y_0, and z_0 are the receiver coordinates; and α and β are the polar and azimuth angles of the receiver direction, respectively (see Figure 1).

In order to calculate U_ω from (3), we should specify the incident electric field. To do this, we introduce for each particular chorus burst the effective source (transmitter) model as a short thin electric dipole at a distance of τ_0 from the receiver along the group velocity resonance cone direction. Indeed, such a dipole effectively radiates quasi-electrostatic

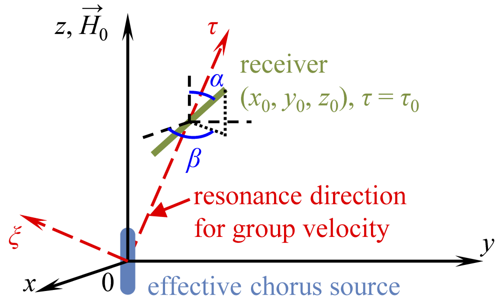

Figure 1. Geometry of the problem (not to scale).

waves, whose electric field is directed almost along the resonance cone (in $\vec{k}$-space) and equals [8]

$$E(\vec{r},t) = -\nabla\Phi(\vec{r},t) = -\frac{i\mu_0^{1/2}e^{-i\pi/4}}{(2\pi)^{3/2}\tau^{1/2}\varepsilon_0} \times \int\limits_{-\infty}^{+\infty}\int\limits_0^{+\infty}\sqrt{k}\Xi(\omega)\rho_{\text{tr}}(\vec{k})|_{\theta=\theta_{\text{res}}} e^{ik\xi+iq(\omega-\omega_0)\tau k-i\omega t}\mathrm{d}\omega\mathrm{d}k, \qquad (6)$$

where τ-axis is directed along the group velocity resonance cone direction, ξ-axis is perpendicular to τ, the transmitter is centered at point $\tau=\xi=0$ (see Figure 1), $\mu_0=\mu(\omega_0)$, $q=(\partial\mu/\partial\omega)|_{\omega=\omega_0}(1+\mu_0^2)^{-1}$, $\mu=\mu(\omega) = \sqrt{|\varepsilon(\omega)/\eta(\omega)|}$, $\Xi(\omega)$ is the spectrum of source time profile, and $\rho_{\text{tr}}(\vec{k})$ is the spectrum of the model charge distribution $\rho_{\text{tr}}(\vec{r})$ along the transmitting dipole. Expression (6) reflects the fact that the field, radiated by a dipole, is the superposition of plane waves having the group velocities predominantly in the direction orthogonal to the resonance surface. Let us choose $\rho_{\text{tr}}(\vec{r})$ in the form that corresponds to a thin dipole of length $2L_{\text{tr}}$ directed along z-axis:

$$\rho_{\text{tr}}(\vec{r}) = -\frac{Q_{\text{tr}}}{2L_{\text{tr}}^2}z\exp\left(-\frac{z^2}{L_{\text{tr}}^2}\right)\delta(x)\delta(y), \qquad (7)$$

where Q_{tr} is the total half-dipole charge amplitude, and $\delta(x)$ is the delta function. Then its spectrum equals

$$\rho_{\text{tr}}(\vec{k})|_{\theta=\theta_{\text{res}}} = \frac{iQ_{\text{tr}}\sqrt{\pi}}{4}kL_{\text{tr}}\cos\theta_{\text{res}}e^{-0.25k^2L_{\text{tr}}^2\cos^2\theta_{\text{res}}}. \qquad (8)$$

This source is chosen for the sake of symmetry and simplicity. We limit ourselves by the dipole approximation and do not consider any multipoles of higher orders because this distribution relatively easy provides the wave field with measured parameters if this field corresponds to a quasi-electrostatic wave packet with a spread in wave vectors. Indeed, half-length L_{tr} of this effective transmitter is determined (by the order of magnitude) by wavenumber k_{obs} that corresponds to the observed spectral maximum:

$$k_{\text{obs}}L_{\text{tr}}\cos\theta_{\text{res}} = 1. \qquad (9)$$

The extremum point equals the characteristic length of $\rho_{\text{tr}}(\vec{k})|_{\theta=\theta_{\text{res}}}$ in (8) so this function, up to a constant factor Q_{tr}, has only one parameter, i.e., $L_{\text{tr}}\cos\theta_{\text{res}}$. This is not necessary satisfied for an arbitrary function which extremum

Table 1. The analyzed chorus events detected by THEMIS.

Event	I	II
THEMIS	C	A
Date (yyyy-mm-dd)	2007-08-28	2008-11-26
UT (hh:mm:ss)	15:51:48	03:18:23
λ_{m} (deg); L	15; 5.4	0; 5.0
ω_0 (s^{-1})	9425	15708
ω_{ce} (s^{-1})	47005	38020
ω_{pe} (s^{-1})	160346	130062
θ_{res} (deg); θ_{obs} (deg)	78.0; 75.0	64.7; 58.0
α (deg)[a]	78.6, 110, 23.7	62.1, 116, 40.1
β (deg)[a]	62.5, 148, 180	51.0, 126, 180
σ[a] ($\tau_0 = \tau_{0\,\max}$)	0.9, 1.6, 0.8	3.2, 3.0, 2.3
σ[a] ($\tau_0 = \tau_{0\,\min}$)	3.2, 5.6, 3.5	10, 9.8, 9.1

[a] The three values correspond to dipoles A, B, and C.

point and characteristic length may differ. However, we use this assumption because it is enough to have one parameter only in order to get the measured wave field parameters from this model. The proposed source model does not need to correspond to a real source of chorus emissions. Its function is only to specify the incident field and to take into account its resonance nature.

According to the aforesaid, the effective length $l_{\mathrm{eff}}(\omega) = |U_\omega(\omega)/[E_\omega(\omega)\cos\Upsilon]|$, where E_ω is the frequency spectrum of the incident field, does not depend on Q_{tr} and $\Xi(\omega)$:

$$l_{\mathrm{eff}}(\omega) = \frac{16\lambda^2}{\delta L_{\mathrm{tr}}^2 L_{\mathrm{rec}} \sin\theta_{\mathrm{res}}} \cdot \frac{\left|\int_0^{2\pi} I_1(\psi,\omega)[\gamma(\psi)]^{-1}\mathrm{d}\psi\right|}{\left|J\left[\frac{5}{4}, A, q(\omega-\omega_0)\tau_0\right]\right|} \quad (10)$$

for the thin rods, and

$$l_{\mathrm{eff}}(\omega) = \frac{16\lambda^2}{\delta L_{\mathrm{tr}}^2 \sin\theta_{\mathrm{res}}} \cdot \frac{\left|\int_0^{2\pi} I_2(\psi,\omega)\mathrm{d}\psi\right|}{\left|J\left[\frac{5}{4}, A, q(\omega-\omega_0)\tau_0\right]\right|} \quad (11)$$

for the small spheres. Here $I_1(\psi,\omega) = 2J[1/4, A, C(\omega)] - J[1/4, A, C(\omega)+\gamma(\psi)L_{\mathrm{rec}}] - J[1/4, A, C(\omega)-\gamma(\psi)L_{\mathrm{rec}}]$; $A = (L_{\mathrm{tr}}\cos\theta_{\mathrm{res}}/2)^2$; $\delta = \sqrt{\varepsilon_0(\varepsilon_0+|\eta_0|)}\cos\Upsilon$; $I_2(\psi,\omega) = J[3/4, A, C(\omega)-\gamma(\psi)L_{\mathrm{rec}}] - J[3/4, A, C(\omega)+\gamma(\psi)L_{\mathrm{rec}}]$; $J(\nu, q_1, q_2) = \Gamma(\nu)\cdot {}_1F_1\left[\nu; 1/2; -q_2^2/(4q_1)\right]/(2q_1^\nu) + iq_2 \times \Gamma(\nu+1/2) \cdot {}_1F_1\left[\nu+1/2; 3/2; -q_2^2/(4q_1)\right]/(2q_1^{\nu+0.5})$; $C(\omega) = q(\omega-\omega_0)\tau_0 - R_0(\psi)$; $\Gamma(\nu)$ and ${}_1F_1(\nu_1;\nu_2;q_1)$ are the gamma function and the confluent hypergeometric function of the first kind, respectively, and $\lambda = c/(f_0\sqrt{g_0})$, $\varepsilon_{xy}(\omega_0) = ig_0$ is the off-diagonal dielectric tensor component. We will consider quasi-monochromatic (at each moment of time) wave packets and choose $\omega = \omega_0$ which simply means an appropriate choice of ω_0 for each spectral component. This does not prevent k to vary in a wide range due to the resonance wave dispersion.

As it follows from the above, the only effective source parameters that determine the receiver effective length are L_{tr} and τ_0. Length L_{tr} is determined by the wave and plasma

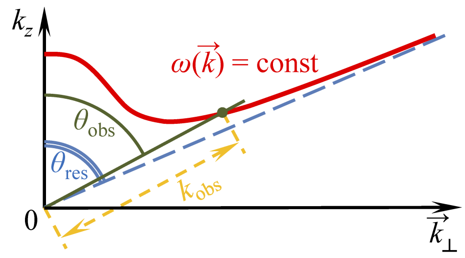

Figure 2. An estimate of k_{obs} from the dispersion relation.

parameters (k_{obs} and θ_{res}) according to (9), and τ_0, generally speaking, is a free parameter and may be varied depending on the physical situation. Therefore, it is important to make an appropriate choice of τ_0 in order to make a reasonable estimate of l_{eff}. Let us discuss this choice. As it was shown in the previous studies using ray tracing [9], the chorus wave normal angle changes significantly due to refraction on the distance corresponding to the geomagnetic latitude λ_{m} change of 1°. Therefore, in order to neglect the refraction effects on the entire transmitter—receiver line and to use (6) for the incident field, we choose τ_0 in the interval $\tau_0 \leq \tau_{0\,\max} = \Lambda$, where Λ corresponds to the geomagnetic latitude change of 0.1° along the geomagnetic field line at given λ_{m} and L-shell. The minimum estimate is obviously determined by the source size: $\tau_{0\,\min} \sim L_{\mathrm{tr}}$.

3 Calculation Results and Discussion

In this section, we apply the obtained formulas to some measurements of chorus wave electric fields onboard THEMIS spacecraft. Each THEMIS spacecraft is equipped with the Electric Field Instrument (EFI) that consists of the three independent orthogonal dipole antennas [10]. These antennas have a half-length of 24.8 m, 20.2 m, and 3.47 m. We will refer to them as dipoles A, B, and C, respectively. Dipoles A and B are straight thin rods, and dipole C consists of 2 small spheres. Their orientation angles α and β have been found from the Flux Gate Magnetometer (FGM) data that provide the results of geomagnetic field measurements [10].

For our analysis we chose 2 bursts (i.e., chorus events; see Table 1) which were previously considered in [6] in relation to electron energization. Values x_0, y_0, and z_0, required for the calculation, were obtained from the wave propagation direction that was found using a singular value decomposition method [11] for the magnetic field waveforms from the Search Coil Magnetometer (SCM) [10]. Value k_{obs} was obtained from the detected wave normal angle θ_{obs} and the dispersion relation (see Figure 2).

The results of calculations are presented in the last two lines of Table 1 where the effective length scale factor $\sigma = l_{\mathrm{eff}}\cos\Upsilon/l_{\mathrm{rec}} = l_{\mathrm{eff}}\cos\Upsilon/(2L_{\mathrm{rec}})$ is shown. One can see that in case $\tau_0 = \tau_{0\,\min}$ the receiver effective length is several times larger as compared to case $\tau_0 = \tau_{0\,\max}$. However, in both cases σ can significantly exceed unity.

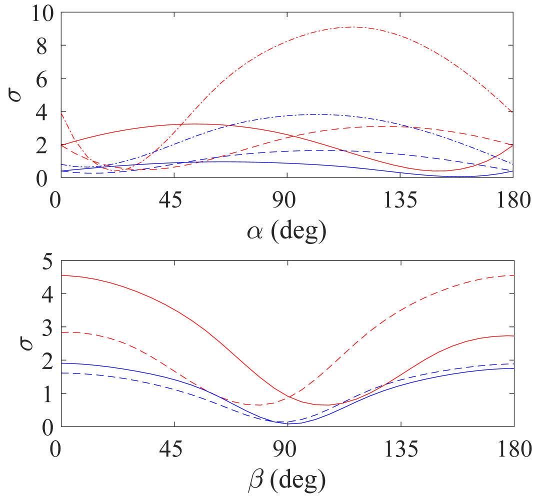

Figure 3. Dependence of the effective length scale factor σ on the antenna orientation angles α (top) and β (bottom). The blue and red lines correspond to the events I and II (see Table 1); the solid, dashed, and dash-dot lines correspond to dipoles A, B, and C, respectively. The fixed values of β and α angles for the top and bottom plots, respectively, are given in Table 1.

Factor σ depends strongly on the receiver orientation with respect to the wave electric field, which in our case is almost parallel to the resonance cone. This dependence has been calculated for the same events in the case $\tau_0 = \tau_{0\max}$ and is shown in Figure 3. As one can see, $\sigma > 1$ and reaches an order of magnitude in many cases of the dipole orientation, with a few exceptions when $\sigma \approx 1$ or $\sigma < 1$. Inequality $\sigma > 1$ means that a dipole quite effectively reradiates quasi-electrostatic wave packets, while $\sigma < 1$ corresponds to the case when the incident wave electric field is quasi-orthogonal to the dipole. In the latter case, electric current is induced inefficiently on the receiver and, hence, the wave reradiation is weak.

We also note that the different cases of the receiver antenna location relative to the real chorus source region are possible. These cases may be modeled—at least, in part—with variation of τ_0. If the waves are excited in a quasi-electrostatic mode and received fairly close to the source region, then it can be appropriate to choose $\tau_0 \sim L_{\mathrm{tr}}$. If the waves are generated in electromagnetic mode and then are transformed into the quasi-electrostatic mode due to refraction, then $\tau_0 \sim \Lambda$. The other possibility is that the receiver is relatively far from the real source region, and refraction is negligible. Then, obviously, $\tau_0 \gg L_{\mathrm{tr}}$, and even inequality $\tau_0 \gg \Lambda$ can be satisfied. In this case, as the calculation results show, $l_{\mathrm{eff}} \gtrsim l_{\mathrm{rec}}$.

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References

[1] Y. V. Chugunov, E. A. Mareev, V. Fiala, and H. G. James, "Transmission of waves near the lower oblique resonance using dipoles in the ionosphere," *Radio Sci.*, vol. 38, 1022, Apr. 2003.

[2] Y. V. Chugunov, V. Fiala, M. Hayosh, and H. G. James, "Whistler mode resonance-cone transmissions at 100 kHz in the OEDIPUS-C experiment," *Radio Sci.*, vol. 47, RS6002, Nov. 2012.

[3] J. Chum, O. Santolík, A. W. Breneman, et al., "Chorus source properties that produce time shifts and frequency range differences observed on different Cluster spacecraft," *J. Geophys. Res. Space Phys.*, vol. 112, A06206, Jun. 2007.

[4] Y. Omura, Y. Katoh, and D. Summers, "Theory and simulation of the generation of whistler-mode chorus," *J. Geophys. Res. Space Phys.*, vol. 113, A04223, Apr. 2008.

[5] O. Santolík, E. Macúšová, I. Kolmašová, et al., "Propagation of lower-band whistler-mode waves in the outer Van Allen belt: Systematic analysis of 11 years of multi-component data from the Cluster spacecraft," *Geophys. Res. Lett.*, vol. 41, pp. 2729–2737, Apr. 2014.

[6] O. V. Agapitov, A. V. Artemyev, D. Mourenas, et al., "The quasi-electrostatic mode of chorus waves and electron nonlinear acceleration," *J. Geophys. Res. Space Phys.*, vol. 119, pp. 1606–1626, Mar. 2014.

[7] Yu. V. Chugunov, E. A. Shirokov, and I. A. Fomina, "On the theory of a short cylindrical antenna in anisotropic media," *Radiophys. Quantum Electron.*, vol. 58, pp. 318–326, Oct. 2015.

[8] E. A. Shirokov and Yu. V. Chugunov, "Nonstationary quasielectrostatic radiation field of dipole antennas in a magnetized plasma in the resonance frequency band," *Radiophys. Quantum Electron.*, vol. 54, pp. 463–473, Dec. 2011.

[9] J. Chum and O. Santolík, "Propagation of whistler-mode chorus to low altitudes: divergent ray trajectories and ground accessibility," *Ann. Geophys.*, vol. 23, pp. 3727–3738, Dec. 2005.

[10] J. Burch and V. Angelopoulos, *The THEMIS Mission*. New York: Springer, 2009.

[11] O. Santolík, M. Parrot, and F. Lefeuvre, "Singular value decomposition methods for wave propagation analysis," *Radio Sci.*, vol. 38, 1010, Feb. 2003.