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Scattering Properties of a Finite Array of Magnetized Plasma Cylinders at the Surface Plasmon Resonances

Alexander V. Ivoninsky*, Alexander V. Kudrin, and Vasiliy A. Es'kin
University of Nizhny Novgorod, Nizhny Novgorod 603950, Russia

Abstract

The scattering properties of a finite array of magnetized plasma cylinders are studied theoretically by using the multiple scattering approach. It is assumed that the array, whose elements are aligned with an external static magnetic field and located equidistantly in free space, is illuminated by an obliquely incident H-polarized plane electromagnetic wave. The far-zone scattering patterns for the array consisting of a relatively large but finite number of cylinders are analyzed in the case where the individual and collective mechanisms of resonance scattering are pronounced in such a system. Numerical calculations performed in the radio-frequency range demonstrate promising prospects for controlling the scattering characteristics of plasma-based arrays near the frequencies of plasmon resonances of their cylindrical elements.

1 Introduction

Over the past decade, the problem of creation of plasma antennas and antenna arrays in the radio-frequency (RF) and microwave ranges has received considerable attention [1–3]. It was established that the electromagnetic characteristics of such systems, which often consist of the plasma-filled tubes, can be similar to the corresponding properties of their metal analogs [1]. However, plasma antennas may have some potential advantages over conventional metal antennas due to ability of controlling their electromagnetic properties by variation in the plasma parameters [2]. In particular, the influence of an external static magnetic field on the radiation pattern of the reconfigurable plasma antenna array has recently been discussed in [3]. It should be mentioned that the most of previous studies on the subject are related to the case where the interaction of electromagnetic radiation with the plasma elements of the array is nonresonant. At the same time, it should be expected that the case where resonant scattering effects are present in the system can be useful in creating the promising plasma-based scattering arrays.

In this work, we analyze theoretically the scattering properties of a two-dimensional array consisting of a finite number of parallel, electrically thin, axially magnetized plasma cylinders, which possess the surface plasmon resonances [4, 5]. For this purpose, we study the problem of resonance scattering of an obliquely incident plane electromagnetic wave by such an array and determine the behavior of the far-zone scattered field. For the sake of brevity, we restrict ourselves to consideration of the special case of an H-polarized incident plane wave, for which the surface plasmon resonances of a single magnetized plasma column are observed for any incidence angle [5].

2 Formulation of the Problem

Consider a planar equidistant array consisting of N identical, infinitely long, uniform circular cylinders of radius a, which are filled with a cold collisionless magnetoplasma and located in free space. The cylinders are aligned with an external static magnetic field $\mathbf{B}_0$, which is parallel to the z axis of a Cartesian coordinate system (x, y, z). The axes of the cylinders, separated by distance L, lie in the xz plane and are specified by the relations $x = jL$ and $y = 0$. Here, $j = -N^{(-)}, \ldots, -1, 0, 1, \ldots, N^{(+)}$, where $N^{(-)}$ and $N^{(+)}$ are the natural numbers satisfying the condition $N^{(+)} + N^{(-)} + 1 = N$. Let a monochromatic H-polarized plane electromagnetic wave with angular frequency ω be incident on the array at an angle θ to the z axis so that the projection of the wave vector $\mathbf{k}$ of this wave onto the xy plane makes an angle ψ with the x axis. The electric and magnetic fields of this wave denoted by the superscript (i) can be written as

$$\mathbf{E}^{(i)} = \mathbf{E}_0^{(i)} \exp[-ik_0(qx\cos\psi + qy\sin\psi + pz)],$$
$$\mathbf{H}^{(i)} = \mathbf{H}_0^{(i)} \exp[-ik_0(qx\cos\psi + qy\sin\psi + pz)]. \quad (1)$$

Hereafter, k_0 is the wave number in free space, $p = \cos\theta$ and $q = \sin\theta$ are the normalized (to k_0) longitudinal, i.e., cylinder-aligned, and transverse components of the wave vector $\mathbf{k}$ in the incident wave, respectively, and the $\exp(i\omega t)$ time dependence is dropped. The vector quantities $\mathbf{E}_0^{(i)}$ and $\mathbf{H}_0^{(i)}$ in the H-polarized plane electromagnetic wave are related by the formula

$$\mathbf{H}_0^{(i)} = Z_0^{-1}(q\cos\psi\mathbf{x}_0 + q\sin\psi\mathbf{y}_0 + p\mathbf{z}_0) \times \mathbf{E}_0^{(i)},$$

where $\mathbf{E}_0^{(i)} = E_0(-\sin\psi\mathbf{x}_0 + \cos\psi\mathbf{y}_0)$, Z_0 is the impedance of free space, and $\mathbf{x}_0$, $\mathbf{y}_0$, and $\mathbf{z}_0$ are the unit vectors of the Cartesian coordinate system. In what follows, all the field quantities will be normalized to the electric-field amplitude E_0. We will consider the case of electrically thin (in terms of the free-space wavelength) cylinders in the array where the inequality $k_0 a \ll 1$ holds.

The cold collisionless magnetoplasma filling the cylinders is described by the dielectric permittivity tensor of standard form [6]:

$$\hat{\varepsilon} = \varepsilon_0 \begin{pmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}. \quad (2)$$

Here, the tensor elements can be written as

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - \omega_H^2},\ g = \frac{\omega_p^2 \omega_H}{\left(\omega^2 - \omega_H^2\right)\omega},\ \eta = 1 - \frac{\omega_p^2}{\omega^2}, \quad (3)$$

where ω_H and ω_p are the gyrofrequency and the plasma frequency of electrons, respectively, and ε_0 is the permittivity of free space. Note that in tensor elements (3), we neglected the contribution of ions, which is possible under the condition $\omega \gg \omega_{\mathrm{LH}}$, which is assumed throughout this work (here, ω_{LH} is the lower hybrid resonance frequency [6]).

The method of solving the above-formulated problem is based on the multiple scattering technique [7], according to which the field scattered by an array of cylindrical objects can be determined if the diffraction characteristics of each element of this array are known. Following the well-known field solution of the corresponding scattering problem for a single magnetized plasma cylinder (see, e.g., [5]), the electromagnetic field can be represented in terms of azimuthal harmonics in a local cylindrical coordinate system (ρ_j, ϕ_j, z) related to the jth cylinder:

$$\mathbf{E} = \sum_{m=-\infty}^{\infty} \mathbf{E}_{j,m} \exp[-i(m\phi_j + k_0 p z)],$$
$$\mathbf{H} = \sum_{m=-\infty}^{\infty} \mathbf{H}_{j,m} \exp[-i(m\phi_j + k_0 p z)], \quad (4)$$

where m is the azimuthal index ($m = 0, \pm1, \pm2, \ldots$). The quantities $\mathbf{E}_{j,m}$ and $\mathbf{H}_{j,m}$ can be expressed via their longitudinal components $E_{j,m;z}(\rho_j)$ and $H_{j,m;z}(\rho_j)$, which satisfy the following equations in the plasma medium [6]:

$$\hat{L}_m E_{j,m;z} - k_0^2 \frac{\eta}{\varepsilon}(p^2 - \varepsilon) E_{j,m;z} = -ik_0^2 \frac{g}{\varepsilon} p Z_0 H_{j,m;z}, \quad (5)$$

$$\hat{L}_m H_{j,m;z} - k_0^2 \left(p^2 + \frac{g^2}{\varepsilon} - \varepsilon\right) H_{j,m;z} = ik_0^2 \frac{g}{\varepsilon} \eta p Z_0^{-1} E_{j,m;z}, \quad (6)$$

where

$$\hat{L}_m = \frac{d^2}{d\rho_j^2} + \frac{1}{\rho_j}\frac{d}{d\rho_j} - \frac{m^2}{\rho_j^2}.$$

In turn, the transverse field components $E_{j,m;\rho}$, $E_{j,m;\phi}$, $H_{j,m;\rho}$, and $H_{j,m;\phi}$ are expressed via the longitudinal components $E_{j,m;z}$ and $H_{j,m;z}$ as

$$E_{j,m;\rho} = A\Big\{ ipg\frac{m}{\rho_j} E_{j,m;z} + ip(\varepsilon - p^2)\frac{dE_{j,m;z}}{d\rho_j} + (\varepsilon - p^2)\frac{m}{\rho_j} Z_0 H_{j,m;z} + gZ_0 \frac{dH_{j,m;z}}{d\rho_j} \Big\}, \quad (7)$$

$$E_{j,m;\phi} = A\Big\{ p(\varepsilon - p^2)\frac{m}{\rho_j} E_{j,m;z} + pg\frac{dE_{j,m;z}}{d\rho_j} - ig\frac{m}{\rho_j} Z_0 H_{j,m;z} - i(\varepsilon - p^2) Z_0 \frac{dH_{j,m;z}}{d\rho_j} \Big\}, \quad (8)$$

$$H_{j,m;\rho} = A\Big\{ \left[g^2 - \varepsilon(\varepsilon - p^2)\right]\frac{m}{\rho_j} Z_0^{-1} E_{j,m;z} - p^2 g Z_0^{-1}\frac{dE_{j,m;z}}{d\rho_j} + ipg\frac{m}{\rho_j} H_{j,m;z} + ip(\varepsilon - p^2)\frac{dH_{j,m;z}}{d\rho_j} \Big\}, \quad (9)$$

$$H_{j,m;\phi} = A\Big\{ ip^2 g\frac{m}{\rho_j} Z_0^{-1} E_{j,m;z} - i\left[g^2 - \varepsilon(\varepsilon - p^2)\right] Z_0^{-1}\frac{dE_{j,m;z}}{d\rho_j} + p(\varepsilon - p^2)\frac{m}{\rho_j} H_{j,m;z} + pg\frac{dH_{j,m;z}}{d\rho_j} \Big\}, \quad (10)$$

where $A = k_0^{-1}\left[g^2 - (p^2 - \varepsilon)^2\right]^{-1}$.

To obtain the corresponding equations for the components of the vector quantities $\mathbf{E}_{j,m}$ and $\mathbf{H}_{j,m}$ in the surrounding medium, one should put $\varepsilon = 1$, $g = 0$, and $\eta = 1$ in Eqs. (5)–(10). Since the expressions for the transverse field components can easily be obtained from Eqs. (7)–(10) if the longitudinal field components are known, we will write explicitly only the expressions for the quantities $E_{j,m;z}$ and $H_{j,m;z}$ inside and outside the cylinders.

The azimuthal harmonics of the longitudinal field components inside the jth plasma column, which are solution of Eqs. (5) and (6), are represented in the form [6]

$$E_{j,m;z}^{(t)} = \frac{i\varepsilon}{pg\eta}\sum_{k=1}^{2} B_{j,m}^{(k)} q_k \times \left(p^2 + q_k^2 + \frac{g^2}{\varepsilon} - \varepsilon\right) J_m(k_0 q_k \rho_j),$$
$$H_{j,m;z}^{(t)} = Z_0^{-1}\sum_{k=1}^{2} B_{j,m}^{(k)} q_k J_m(k_0 q_k \rho_j). \quad (11)$$

Here, the superscript (t) denotes the field transmitting to the cylinder, J_m is a Bessel function of the first kind of order m, $B_{j,m}^{(1)}$ and $B_{j,m}^{(2)}$ are the amplitude coefficients corresponding to the azimuthal index m for the field inside the jth cylinder, and $q_k(p)$ stands to denote the normalized (to k_0) transverse wave numbers of two normal waves ($k = 1, 2$) in a magnetoplasma. Expressions for $q_k(p)$ can be found elsewhere [5, 6]. The presence of two transverse wave numbers q_1 and q_2 in a magnetoplasma, which correspond to the same longitudinal wave number p, is related to anisotropic properties of a magnetized plasma medium, in which two normal waves, ordinary and extraordinary, are simultaneously excited by an obliquely incident plane electromagnetic wave.

The field outside the plasma cylinders is a superposition of the incident-wave and scattered fields. The azimuthal harmonics of the longitudinal components of the field in the incident H-polarized plane wave in the local coordinates of the jth cylinder are written as

$$E^{(i)}_{j,m;z} = 0,$$
$$H^{(i)}_{j,m;z} = Z_0^{-1}(-i)^m qJ_m(k_0 q\rho_j)e^{i(m\psi - k_0 qLj\cos\psi)}. \quad (12)$$

The field scattered by each cylinder, which is denoted by the superscript (s), can also be written in terms of cylindrical functions, and the longitudinal components of its azimuthal harmonics have the form

$$E^{(s)}_{j,m;z} = D^{(E)}_{j,m} qH^{(2)}_m(k_0 q\rho_j),$$
$$H^{(s)}_{j,m;z} = Z_0^{-1} D^{(H)}_{j,m} qH^{(2)}_m(k_0 q\rho_j), \quad (13)$$

where $H^{(2)}_m$ is a Hankel function of the second kind of order m, and $D^{(E)}_{j,m}$ and $D^{(H)}_{j,m}$ are the scattering coefficients of jth cylinder, which correspond to the azimuthal index m.

Using the standard method based on Graf's addition theorem for cylindrical functions, the field scattered by all the plasma cylinders of the array can be expressed in terms of azimuthal harmonics in the local coordinate system of the jth cylinder [7]. Next, satisfying the boundary conditions for the tangential field components on the surface of this scatterer and using the known scattering matrix of a single magnetized plasma cylinder, we can exclude the coefficients $B^{(1,2)}_{j,m}$ and obtain a system of equations for the scattering coefficients in the form

$$\left(\hat{S}^{-1}_m\right)_{11} D^{(E)}_{j,m} + \left(\hat{S}^{-1}_m\right)_{12} D^{(H)}_{j,m} = \sum_{n=-\infty}^{\infty}\sum_{l\neq j} h^{(j)}_{l,n-m} D^{(E)}_{l,n},$$
$$\left(\hat{S}^{-1}_m\right)_{21} D^{(E)}_{j,m} + \left(\hat{S}^{-1}_m\right)_{22} D^{(H)}_{j,m} = \sum_{n=-\infty}^{\infty}\sum_{l\neq j} h^{(j)}_{l,n-m} D^{(H)}_{l,n}$$
$$+(-i)^m \exp[i(m\psi - k_0 qLj\cos\psi)]. \quad (14)$$

Here, $h^{(j)}_{l,n-m} = H^{(2)}_{n-m}(k_0 qL|l-j|)$ for $l < j$ and $h^{(j)}_{l,n-m} = (-1)^{n-m} H^{(2)}_{n-m}(k_0 qL|l-j|)$ for $l > j$, whereas the notation $\left(\hat{S}^{-1}_m\right)_{ik}$ is used for the corresponding element $(i,k = 1,2)$ of the matrix $\hat{S}^{-1}_m$, which is the inverse of the scattering matrix $\hat{S}_m$ for a single cylinder. The elements of the matrix $\hat{S}_m$ are expressed via the scattering coefficients $D^{(E)}_m$ and $D^{(H)}_m$ of a single cylinder [5] and written as $\left(\hat{S}_m\right)_{11} = D^{(E)}_m$ and $\left(\hat{S}_m\right)_{21} = D^{(H)}_m$ in the case of illumination by an E-polarized plane wave, and as $\left(\hat{S}_m\right)_{12} = D^{(E)}_m$ and $\left(\hat{S}_m\right)_{22} = D^{(H)}_m$ if an H-polarized plane wave is incident on the cylinder. Since the expressions for the elements $\left(\hat{S}_m\right)_{ik}$ turn out to be very cumbersome, they are not given here for the sake of brevity. Next, restricting ourselves to consideration of only azimuthal harmonics with the indices $0, \pm 1, \ldots, \pm M$ and bearing in mind that the array contains N cylindrical scatterers, we arrive from Eq. (14) at the system of $2N(2M+1)$ algebraic equations, which allow us to determine numerically the desired coefficients $D^{(E)}_{j,m}$ and $D^{(H)}_{j,m}$ for all the cylinders in the array. The quantity M, which characterizes the number of the azimuthal harmonics to be taken into account, is determined from the numerical tests for the accuracy of the solution obtained. The quantities $B^{(1,2)}_{j,m}$ can be expressed via $D^{(E,H)}_{j,m}$ if required.

To analyze the behavior of the far-zone scattered field (in the xy plane), we will calculate the scattering pattern $\sigma(\phi)$. In cylindrical coordinate system (ρ, ϕ, z) related to the $j = 0$ element of the array, this quantity can be expressed as

$$\sigma(\phi) = \lim_{\rho\to\infty} \rho S^{(s)}_\rho(\rho, \phi)/|\mathbf{S}^{(i)}|. \quad (15)$$

Here, $S^{(s)}_\rho$ is the radial component of the time-averaged Poynting vector $\mathbf{S}^{(s)} = (1/2)\mathrm{Re}\left(\mathbf{E}^{(s)} \times \mathbf{H}^{(s)*}\right)$ of the scattered field in the xy plane and $|\mathbf{S}^{(i)}|$ is the magnitude of this vector in the incident wave.

3 Numerical Results

Numerical calculations of the scattering characteristics of the array were performed for the following values of dimensionless parameters: $\omega_p/\omega_H = 8.02$, $\omega_p a/c = 0.188$, $N = 25$ ($N^{(\pm)} = 12$), $\theta = \pi/4$, and $0 < \psi \leq \pi/2$. Recall that the analysis of the quantities $\left(\hat{S}_m\right)_{ik}$ for a single cylindrical plasma element in the case $k_0 a \ll 1$ shows the presence of plasmon resonances [4, 5], at which enhanced scattering occurs from the cylinder. The surface-plasmon resonances corresponding to the azimuthal harmonics $m = \pm 1$ have the widest linewidths compared with other plasmon resonances and are the most important in the case considered. The frequencies of these dipole-type resonances, which depend on the plasma parameters ω_p and ω_H and the incidence angle θ in a certain manner [5], will be denoted as $\omega_{\mathrm{SP},\pm 1}$. At the same time, an equidistant array of parallel cylinders has the so-called Rayleigh–Wood diffraction anomalies at the frequencies hereafter denoted as $\omega^{(\pm)}_n$ $(n = 1, 2, \ldots)$. In the limiting case of an infinite array, these frequencies as functions of the angles θ and ψ and the inter-cylinder distance L can be found from the relationship $\omega^{(\pm)}_n L\sin\theta(1 \mp \cos\psi)/c = 2\pi n$. It is evident that for $\psi = \pi/2$, we have $\omega^{(+)}_n = \omega^{(-)}_n = \omega_n$.

As follows from numerical calculations, the most intriguing scattering patterns $\sigma(\phi)$ are observed if the wave frequency is simultaneously close to the frequency $\omega^{(\pm)}_n$ and one of the surface-plasmon resonance frequencies $\omega_{\mathrm{SP},\pm 1}$ of a single cylinder. In this case, the behavior of the far-zone field can change qualitatively with a relatively small variation in any of the parameters of the problem. This situation is shown in Fig. 1 for $\psi = \pi/2$ in the case where the frequency ω lies near the frequencies $\omega_{\mathrm{SP},1}$ and $\omega_1 = 0.958\omega_{\mathrm{SP},1}$, being closer to either ω_1 or $\omega_{\mathrm{SP},1}$ [Figs. 1(a) and 1(b), respectively]. The normalized (to maximum value) scattering pattern $\sigma(\phi)$ for $\omega/\omega_1 = 0.988$ in Fig. 1(a) has two narrow lobes at $\phi = 90°$ and $\phi = 270°$, which correspond to the transmitted and reflected fields, and two wide side lobes in the directions $\phi = 0$ and $\phi = 180°$, which are perpendicular to the incidence plane $x = 0$ and lie in the array plane. At the same time, for $\omega/\omega_1 = 1.029$, the pattern $\sigma(\phi)$ in Fig. 1(b) has a greater number of the side lobes which correspond to the scattered waves propagating at certain angles from the array plane.

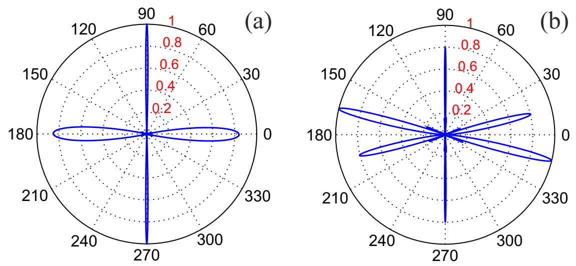

Figure 1. Normalized scattering pattern $\sigma(\phi)$ for an array consisting of $N = 25$ ($N^{(\pm)} = 12$) elements at $k_0 L \sin\theta/2\pi = 0.988$ (a) and $k_0 L \sin\theta/2\pi = 1.029$ (b) if $L/a = 65$, $\theta = \pi/4$, and $\psi = \pi/2$.

Another interesting situation can take place for $\psi \neq \pi/2$ when $\omega_n^{(-)} \neq \omega_n^{(+)}$. In this case, it is possible that at fixed values of θ and L, the incident-wave frequency is again close to the surface-plasmon resonance frequency $\omega_{\mathrm{SP},1}$ and lies near the array frequencies $\omega_n^{(-)}$ and $\omega_{\tilde{n}}^{(+)}$, which correspond to different diffraction orders n and $\tilde{n}$. Since the values of $\omega_n^{(-)}$ and $\omega_{\tilde{n}}^{(+)}$ depend on the incidence angle ψ, the frequency ω can become closer to either of these array frequencies during the very small variation in ψ. The corresponding changes in the scattering patterns $\sigma(\phi)$ at $\omega/\omega_{\mathrm{SP},1} = 0.994$, which are caused by appearance or disappearance of the side lobes near the directions $\phi = 0$ and $\phi = 180°$, are demonstrated in Figs. 2(a) and 2(b) for $\omega = 1.019\omega_1^{(+)} = 0.937\omega_3^{(-)}$ and $\omega = 0.814\omega_1^{(+)} = 1.006\omega_3^{(-)}$, respectively.

4 Conclusion

In this work, we have studied the scattering of an H-polarized plane electromagnetic wave by an equidistant array of parallel cylinders filled with a cold collisionless magnetoplasma. The far-zone scattering patterns have been analyzed in the special cases where both the surface plasmon resonances of the array elements and the collective multiple scattering resonances are present in such a system. The results obtained demonstrate the capabilities of controlling the scattering characteristics of the plasma-based arrays by

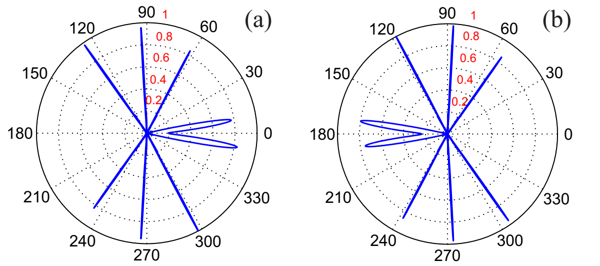

Figure 2. Normalized scattering pattern $\sigma(\phi)$ for an array consisting of $N = 25$ ($N^{(\pm)} = 12$) elements at $\psi = 0.345\pi$ (a) and $\psi = 0.305\pi$ (b) if $k_0 L \sin\theta/2\pi = 1.915$, $L/a = 120$, and $\theta = \pi/4$.

switching between the different regimes of resonance scattering.

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References

[1] I. Alexeff, T. Anderson, S. Parameswaran, E. P. Pradeep, J. Hulloli, and P. Hulloli, "Experimental and theoretical results with plasma antennas," *IEEE Trans. Plasma Sci.*, **34**, 2, April 2006, pp. 166–172, doi:10.1109/TPS.2006.872180.

[2] T. Anderson, *Plasma Antennas*, London, UK: Artech House, 2011.

[3] A. D. J. Fernandez-Olvera, D. Melazzi, and V. Lancelotti, "Beam-forming and beam-steering capabilities of a reconfigurable plasma antenna array," *Prog. Electromagn. Res. C*, **65**, June 2016, pp. 11–22, doi:10.2528/PIERC16031506.

[4] V. A. Es'kin, A. V. Ivoninsky, A. V. Kudrin, and C. Krafft, "Poynting vector behaviour during the resonance scattering of a plane electromagnetic wave by a gyrotropic cylinder," *Phys. Scr.*, **91**, 1, January 2016, p. 015502, doi:10.1088/0031-8949/91/1/015502.

[5] V. A. Es'kin, A. V. Ivoninsky, and A. V. Kudrin, "Scattering of an obliquely incident plane electromagnetic wave by a magnetized plasma column: Energy flow patterns at plasmon resonances," *Prog. Electromagn. Res. B*, **63**, September 2015, pp. 173–186, doi:10.2528/PIERB15080402.

[6] I. G. Kondrat'ev, A. V. Kudrin, and T. M. Zaboronkova, *Electrodynamics of Density Ducts in Magnetized Plasmas*, Amsterdam, The Netherlands: Gordon and Breach, 1999.

[7] K. Yasumoto, *Electromagnetic Theory and Applications for Photonic Crystals*, Boca Raton, FL: Taylor and Francis, 2006.