Design Strategies for Metamaterial Quadrature Power Dividers in CP Antennas: Are Two CRLH-Loaded Lines Necessary?

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Abstract

Integrating composite right/left-handed (CRLH) transmission lines into quadrature power dividers (QPDs) has demonstrated potential for enhanced bandwidth in circularly-polarized antenna applications. Despite the excitement about CRLH, optimal design strategies remain ambiguous, and previous work proposed varied embodiments of CRLH integrations but do not direct designers towards optimal designs strategies for QPDs. This paper investigates the necessity to integrate CRLH in both output paths of QPDs versus one output path. Our analysis reveals that there is no advantage from a dispersion perspective. Both analytical models and simulations agree with this conclusion, encouraging future users of this concept to employ a single CRLH line approach.

1 Introduction

The last two decades have sparked significant interest in metamaterials, which are often described as effective materials having properties not exhibited by naturally-occuring materials. Among various embellishments of metamaterials, composite right/left-handed transmission lines (CRLH-TL's) have been directly integrated into microwave circuits to develop interesting, high-performance devices. This class of metamaterials enables the direct realization of left-handed properties within typical guided-wave structures using non-resonant loading elements.

Numerous devices have integrated CRLH-TLs into their physical operation to achieve enhanced bandwidth, miniaturization, or even dual-band performance. Some examples include phase shifters [1], quadrature hybrids [2], rat-race hybrids [3], baluns [4, 5], quadrature power dividers [6, 7], among others. For circularly-polarized (CP) antenna applications, wideband quadrature power dividers have very attactive properties. The most attractive property for these applications is the CRLH ability to create nearly constant phase-shifts over wide bandwidths. The CRLH-based phase shifter can be directly integrated into any CP antenna whose CP feature is created by properly exciting and phasing multiple polarizations. While the application of these CRLH quadrature power-dividers has been demonstrated in previous literature, a full study on different design strategies has not been divulged.

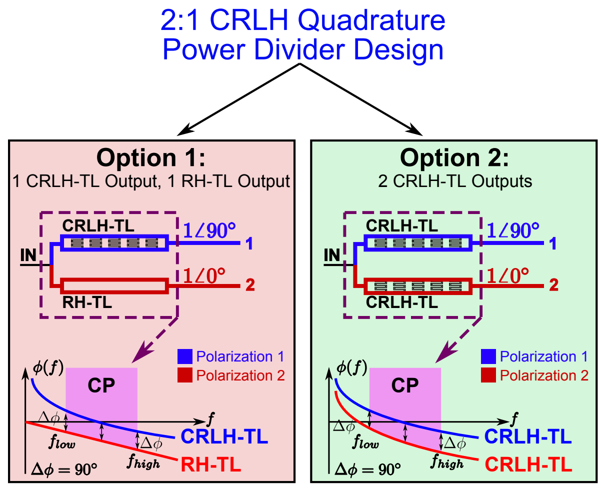

Figure 1. Two strategies using CRLH-TLs that create wideband phase shifters. This study reveals that only one CRLH line (option 1) is necessary to reap the benefits.

This particular study focuses attention on the question of whether a CRLH-based power divider should integrate CRLH on *all* of its outputs or only a subset of the outputs. This is nicely illustrated in Fig. 1, where designers have two options for developing a CRLH-based power divider giving a wideband phase shift of 90°. This particular figure focuses on two design strategies for 1-to-2 power dividers, where Option 1 features one output loaded with CRLH and another output created from a simple right-handed transmission-line (RH-TL). Option 2, on the other hand, divides power to two output branches, where both contain CRLH-TLs. Both option 1 and option 2 were previously investigated for broadband baluns in [4] and [5], respectively, but a deeper investigation between the two approaches was never attempted to the authors' best knowledge. Option 2 had previously been suggested to offer more bandwidth because the two dispersion curves appear to share a similar shape. It is not immediately obvious, however, if two CRLH-TL outputs would provide enhanced performance over the case where 1 RH-TL and 1 CRLH-TL outputs were utilized. This is an important result for both designers and from gaining deeper insights into the dispersion engineering of these structures. We begin this investigation by analyzing the two options from the effective medium models, followed by a lumped element analysis. Our findings reveal that only one CRLH-TL line is necessary, and having two CRLH-TLs is redundant in the optimal design of phase quadrature power dividers. While adding LH features in each output gives more design flexibility, the use of more lumped components can increase cost and RF losses.

2 Theoretical Evaluation using Effective Medium Models

The objective in our application is to achieve a nearly constant 90° phase difference between the two perpendicular polarizations radiated from an antenna, which requires a wideband QPD like the ones seen in Fig. 1. In both option 1 and option 2, we are manipulating the dispersion of the transmission line to achieve the benefit of wideband, constant-phase-difference outputs. The dispersion in each case is illustrated in Fig. 1, where the effective "zero-phase" frequency can be controlled for either one or both outputs. To understand the fundamental limitations of each approach, we first analyze both options assuming that the lines can be modeled by an effective RH or CRLH medium.

2.1 Option 1: One RH-TL and One CRLH-TL Output Paths

For the case of option 1, we can analyze the output phase of both output paths. Fig. 2 illustrates the effective medium model for the QPD option 1, where both the RH-TL and the CRLH-TL have infinitesimal reactances that contribute to the transmission line features. The CRLH-TL path has a propagation constant given as $\beta_1(\omega) = \beta_R(\omega) + \beta_L(\omega)$ and a length ℓ_1, where the subscripts R and L signify contributions from the right and left-handed portions of the line [8]. The RH-TL path has a propagation constant given

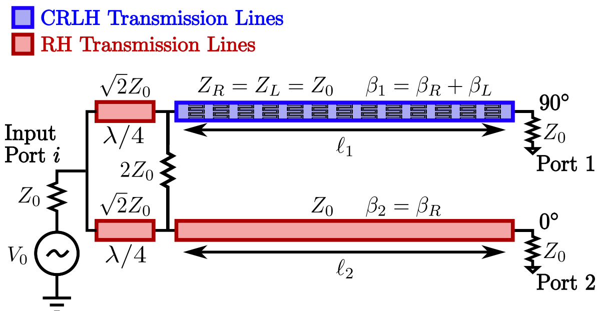

Figure 2. Effective medium model for QPD option 1: one RH-TL and one CRLH-TL output paths.

as $\beta_2(\omega) = \beta_R(\omega)$ and a length ℓ_2, where there is only a right-handed contribution to the propagation constant. The CRLH-TL line adds the following phase shift to a voltage wave exiting the power divider as

$$\phi_1 = -\beta_1(\omega)\ell_1 = -\omega\ell_{R1}\sqrt{L'_R C'_R} + \frac{\ell_{L1}}{\omega\sqrt{L'_L C'_L}} \quad (1)$$

where L'_R and C'_R are the right-handed line inductance and capacitance while L'_L and C'_L represent the left-handed "times-unit-length" inductance and capacitance [8]. The CRLH-TL phase shift can also be broken down into the right-handed and left-handed phase shifts as $\phi_1 = -\beta_R(\omega)\ell_{R1} - \beta_L(\omega)\ell_{L1}$. This is helpful when comparing both options. The phase shift for a wave exiting on the RH-TL output branch can be written as

$$\phi_2 = -\beta_R(\omega)\ell_2 = -\omega\ell_{R2}\sqrt{L'_R C'_R}. \quad (2)$$

where it should be noticed that the line inductance and capacitance are the same as those in output 1 by closely comparing (1). We can safely assume this because the implementation of the RH-TL and the CRLH-TL will result in the right-handed portions having the same characteristic impedance Z_0 (to meet impedance matching and CRLH balanced conditions) and the same phase velocity $v_p = 1/\sqrt{L'_R C'_R}$. Ultimately, this means that the right-handed propagation constant can be assumed to be equal for both outputs, while the length of the right-handed portions will determine their contributions to the phase.

Quadrature phase is achieved through the phase difference between the two outputs. This phase difference can be found as

$$\Delta\phi_1 = \phi_1 - \phi_2 = \omega(\ell_{R2} - \ell_{R1})\sqrt{L'_R C'_R} + \frac{\ell_{L1}}{\omega\sqrt{L'_L C'_L}} \quad (3)$$

which can recast as

$$\Delta\phi_1 = \omega\Delta\ell_R\sqrt{L'_R C'_R} + \frac{\ell_{L1}}{\omega\sqrt{L'_L C'_L}} \quad (4)$$

From a practical perspective, we have the capability to adjust the difference in right-handed length $\Delta\ell_R = \ell_{R2} - \ell_{R1}$ and the product $\ell_L/\sqrt{L'_L C'_L}$. This would be done, for example, by adjusting the length of microstrip lines to control $\Delta\ell_R$. The product $\ell_L/\sqrt{L'_L C'_L}$ can be controlled by changing the LH inductance and capacitance (while still ensuring that $\sqrt{L'_L/C'_L} = Z_0$) or by changing the length of the left-handed portion of the line. This is accomplished by modifying lumped element capacitances and inductances along with the number of unit cells.

2.2 Option 2: Two CRLH-TL Output Paths

We can use a nearly identical analysis for option 2, where we have two output paths with CRLH-TL features. The effective medium model is illustrated in Fig. 3, where we

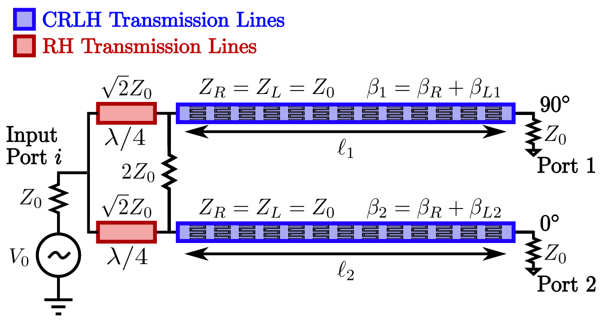

Figure 3. Effective medium model for QPD option 2: two CRLH-TL output paths.

have two balanced CRLH-TL lines having a characteristic impedance Z_0, propagation constants of $\beta_1(\omega) = \beta_R(\omega) + \beta_{L1}(\omega)$ and $\beta_2(\omega) = \beta_R(\omega) + \beta_{L2}(\omega)$, and line lengths of ℓ_1 and ℓ_2, respectively. Just as in option 1, we are assuming that the RH portion of the propagation constants β_R is the same for both output paths.

Like option 1, the phase difference between the outputs can be calculated by

$$\Delta\phi_2 = \Phi_1 - \Phi_2 = \\ \omega \Delta\ell_R \sqrt{L'_R C'_R} + \frac{1}{\omega}\left(\frac{\ell_{L1}}{\sqrt{L'_{L1}C'_{L1}}} - \frac{\ell_{L2}}{\sqrt{L'_{L2}C'_{L2}}}\right) \quad (5)$$

where L'_{L1}, C'_{L1} and L'_{L2}, C'_{L2} are the "times-unit-length" inductances and capacitances for outputs 1 and 2, respectively. The subscript 2 in $\Delta\phi_2$ simply denotes that this is the phase difference for option 2. Note also $\Delta\ell_R = \ell_{R2} - \ell_{R1}$.

Comparing (4) and (5) demonstrates several important features between options 1 and 2. Both equations share the same $A\omega + B/\omega$ frequency dependence, where A and B represent constants of proportionality. In fact, if these coefficients can be tuned to the same values, then both options can achieve identical results, which is the basis of our assertion that option 2 offers no further dispersion shaping.

Tuning these coefficients A and B is easily accomplished. If we assume that similar waveguide structures would be used to create the power dividers (for microstrip lines, this means identical substrates), then the first coefficient $A = \Delta\ell_R \sqrt{L'_R C'_R}$ will be identical between options 1 and 2. It is the left-handed portion of the line that requires some justification, but that is also fairly obvious after close examination of (4) and (5). For simplicity, we can safely assume that at least one of the paths will have identical left-handed features, e.g. $L'_{L1} = L'_L$ and $C'_{L1} = C'_L$. We then recast (5) as

$$\Delta\phi_2 = \omega \Delta\ell_R \sqrt{L'_R C'_R} + \frac{1}{\omega}\frac{\Delta\ell_L}{\sqrt{L'_{L1}C'_{L1}}} \quad (6)$$

where

$$\Delta\ell_L = \ell_{L1} - f_v \ell_{L2} \quad (7)$$

$$f_v = \frac{v_{p,L2}}{v_{p,L1}} = \frac{\omega^2\sqrt{L'_{L1}C'_{L1}}}{\omega^2\sqrt{L'_{L2}C'_{L2}}} \quad (8)$$

The left-handed length difference $\Delta\ell_L$ contains a ratio f_v which is used to describe the effective length of ℓ_{L2} in the LH medium described by L'_{L1}, C'_{L1}. This can be nicely done by taking the ratio of the phase velocity $v_p = \omega/\beta$ between the two left-handed mediums. The ratio describes how much longer/shorter the LH portion of output path 2 compares when using a common LH reference medium.

This analysis highlights our ultimate finding: the phase response is unaffected by choosing option 1 or 2. Even in the case where both outputs are loaded with CRLH, it is the path length *differences* ($\Delta\ell_R$, $\Delta\ell_L$) that gives us the phase response. Based on (4) and (6), we could set $\ell_{L2} = 0$ and still get the dispersion we desire. In other words, any extra LH features that we put in output path 2 is superfluous and does not provide any new dispersion-shaping capabilities. One difference is that designers are given a little more freedom in choosing the inductance/capacitance values for the LH portions of the line. However, this comes at the cost of more lumped components, which can increase losses.

3 Validation using Lumped Element Models

In this section, we evaluate several practical QPD designs for both options 1 and 2 to validate our analysis and justify our observations. We arbitrarily choose a center frequency of $f_0 = 2.4$ GHz, but the conclusions are invariant with frequency. We simulate these designs using Keysight ADS simulation suite. We also use the same Wilkinson power divider throughout all simulations. We discuss both a simple design procedure for each option and provide relevant examples. In each case, we assume to design the structures using the two frequency design approach similar to those in [2, 6], where (4) and (5) are set equal to $\Delta\phi' = \pi/2$ for $f_1 = f_0 - \Delta f/2$ and $f_2 = f_0 + \Delta f/2$. The fractional bandwidth was set to 50% (similar to our work in [7]), providing values of $f_1 = 1.8$ GHz and $f_2 = 3.0$ GHz.

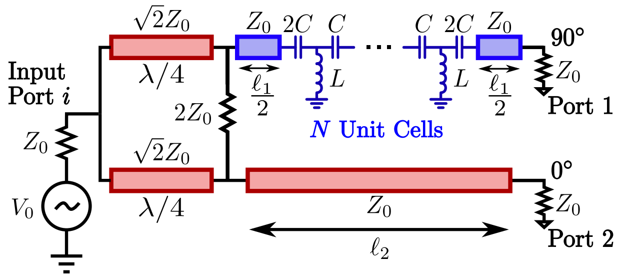

Figure 4. Lumped element model for QPD option 1, where the left-handed portions of the CRLH-TL are implemented using lumped capacitors and inductors.

We implement the RH portions of the line with TEM transmission lines and the LH portions using lumped elements. The lumped element models for both options are illustrated in Figs. 4-5. Using finite sized lumped elements will exhibit some deviation from the effective medium model, but we can approximately relate the lumped element values back to the effective mediums. We can find the values for the lumped element inductors/capacitors by substituting $\ell_L = N\Delta z$, $L = L'_L/\Delta z$, and $C = C'_L/\Delta z$ in (4) and (5).

We designed several QPDs for comparison. We started every design by assuming a certain number of unit cells and LH ratio f_v, and computing the resulting lumped element values to achieve 90° phase difference. In an attempt to keep things similar between options 1 and 2, we also assume that number of LH unit cells in path 1 is the same for both options, i.e. $N = N_1$.

Fig. 6 reveals the phase performance for the case where $N = N_1 = 5$. We tested several different designs for op-

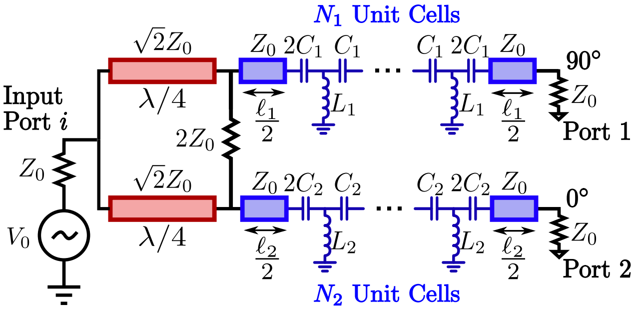

Figure 5. Lumped element model for QPD option 2, where the left-handed portions of the CRLH-TL are implemented using lumped capacitors and inductors.

tion 2, where we chose a different number of LH unit cells N_2 for path 2. Clearly the phase performance is nearly identical in all cases, which highlights the fact that adding LH features in both paths is not necessary. Once we design the dispersion to satisfy $\Delta\phi = \pi/2$ for f_1 and f_2, the frequency dispersion will be nearly identical between option 1 and 2. We also tested this finding for small numbers of unit cells, i.e. $N = N_1 = 2$, where the results are shown in Fig. 7. The only differences occur at the lower end of the frequency bands shown. This is because the finite lumped element behavior starts to dominate, and the effective medium model becomes inaccurate. As a final note, these QPDs offer tight phase control with a phase difference of $\Delta\phi = 88.5 \pm 1.5°$ over a 50% bandwidth, which is very useful for low AR circularly-polarized antennas [7].

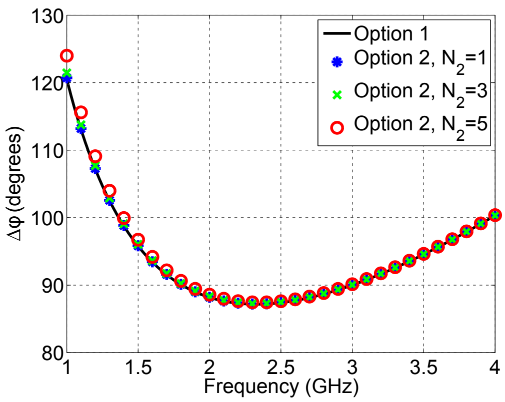

Figure 6. Performance for five cascaded unit cells in path 1, where $N = N_1 = 5$, $f_v = 2$, and $Z_0 = 50\Omega$. We vary the number of unit cells in path 2 (N_2) and redesign each case.

4 Conclusion

This paper uncovers an important observation that will impact design decisions for CRLH quadrature power dividers (and even arbitrary phase-shifting CRLH power dividers in general). The observation is that adding LH features to both output lines does *not* enhance the dispersion shaping capabilities. As we prove analytically in this paper, this is because the phase difference is only dependent on the RH and LH *path length differences*. This is very useful because it simplifies the design and reduces the number of lumped elements needed, potentially reducing both cost and losses incurred by these elements.

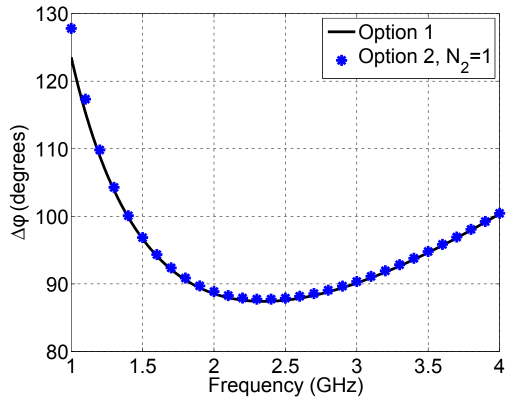

Figure 7. Performance for two cascaded unit cells in path 1, where $N = N_1 = 2$, $f_v = 2$, and $Z_0 = 50\Omega$. The only possible value for N_2 was $N_2 = 1$, otherwise zero or negative capacitance values were found.

References

[1] M. A. Antoniades and G. V. Eleftheriades, "Compact linear lead/lag metamaterial phase shifters for broadband applications," *IEEE Antennas Wireless Propag. Lett.*, **2**, 1, pp. 103–106, 2003.

[2] I.-H. Lin, M. DeVincentis, C. Caloz, and T. Itoh, "Arbitrary dual-band components using composite right/left-handed transmission lines," *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 4, pp. 1142–1149, 2004.

[3] H. Okabe, C. Caloz, and T. Itoh, "A compact enhanced-bandwidth hybrid ring using an artificial lumped-element left-handed transmission-line section," *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 3, pp. 798–804, 2004.

[4] C.-J. Lee, K. Leong, and T. Itoh, "Broadband microstrip-to-cps and microstrip-to-cpw transitions using composite right/left-handed metamaterial transmission lines," *IEE Proceedings-Microwaves, Antennas and Propagation*, vol. 153, no. 3, pp. 241–246, 2006.

[5] M. Antoniades and G. V. Eleftheriades, "A broadband wilkinson balun using microstrip metamaterial lines," *IEEE Antennas Wireless Propag. Lett.*, vol. 4, pp. 209–212, 2005.

[6] C.-H. Tseng and C.-L. Chang, "A broadband quadrature power splitter using metamaterial transmission line," *IEEE Microw. Wireless Compon. Lett.*, vol. 18, no. 1, pp. 25–27, 2008.

[7] J. M. Kovitz, Y. Rahmat-Samii, and J. Choi, "Dispersion engineered right/left-handed transmission lines enabling near-octave bandwidths for wideband CP patch arrays," in *IEEE Int. Symp. Anten. Prop.*, 2015, pp. 2525–2526.

[8] A. Lai, T. Itoh, and C. Caloz, "Composite right/left-handed transmission line metamaterials," *IEEE Microw. Mag.*, **5**, no. 3, pp. 34–50, 2004.