2nd URSI AT-RASC, Gran Canaria, 28 May – 1 June 2018

A Sheath Impedance Model for the Van Allen Probes EFW Instrument

David P. Hartley* (1), Craig A. Kletzing(1), William S. Kurth(1), George B. Hospodarsky(1), Scott R. Bounds(1),
Terrance F. Averkamp(1), John W. Bonnell(2), Ondrej Santolik(3,4), and John R. Wygant(5)
(1) Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa, USA.
(2) Space Sciences Laboratory, University of California, Berkeley, CA, USA.
(3) Department of Space Physics, Institute of Atmospheric Physics, Prague, Czech Republic.
(4) Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic.
(5) School of Physics and Astronomy, University of Minnesota, Minneapolis, MN, USA.

Abstract

A technique to quantitatively determine the variable coupling impedance between the Van Allen Probes Electric Field and Waves (EFW) instrument and the ambient magnetospheric plasma is presented. This is achieved by applying the cold plasma dispersion relation to whistler-mode chorus waves and plasmaspheric hiss. It is then possible to perform comparisons between the electric field wave power spectra predicted by cold plasma theory (using magnetic field observations), and the electric field wave power measured by the EFW spherical double probes instrument. Investigation of the ratio between observed and calculated wave powers, as a function of frequency and plasma density, reveals a structure consistent with signal attenuation via the formation of a plasma sheath around the electric field sensors. Further analysis reveals that anomalous gains can occur at specific densities and frequencies due to the shorter spin-axis antennas measuring too much electric field. Antenna shorting effects are also apparent in the low-density regime. A density-dependent model is developed in order to quantify these effects. This sheath impedance model allows for the sheath resistance, sheath capacitance, and relative effective antenna length to be quantified at any density frequently encountered on-orbit and is demonstrated to be successful in significantly improving agreement between calculated and observed power spectra and wave powers.

1. Introduction

Obtaining high-accuracy observations of the wave electric field in the near-Earth environment is an important step towards gaining a more complete understanding of the physical processes that accelerate charged particles in the inner magnetosphere. Similar to the Electric Field and Waves (EFW) instrument [1] onboard the Van Allen Probes spacecraft, multiple spacecraft missions, both past and present, implement spherical double probe sensors to make observations of both the DC and wave electric field. Due to the spinning Van Allen Probes spacecraft, the electric field in the spin-plane is measured by centrifugally deployed wire booms, consisting of fine wire and spherical sensors achieving a probe separation of ~100m. For observations in the spin axis, rigid booms are required and provide a significantly shorter separation of ~14m between the spherical double probe sensors. The coupling impedance of the sensor-plasma interface varies with ambient plasma conditions [2, 3, 4] leading to a frequency-dependent response function that is dependent on the on-orbit conditions and is therefore not precisely known. Sheath impedance functions also depend upon the antenna/sensor properties, such as probe separation and sensor geometry. As such, electric field wave measurements have always been subject to some degree of uncertainty. Here, we present a technique of using EMFISIS [5] magnetic field observations and cold plasma theory to quantify the antenna-sheath impedance and its effect on the total electric field wave power observed by the Van Allen Probes EFW instrument.

Due to the varying antenna and sensor properties used for spin plane and spin axis observations, the sheath impedance functions are different for each direction. However, it can be demonstrated that despite the different antenna types, an accurate representation of the sheath impedance of the total electric field wave power (sum over all antennas) can be obtained using a single sheath impedance function, assuming a random contribution from each measurement direction. Since antenna and sensor properties vary between instruments and spacecraft, the sheath impedance functions reported here are only valid for the Van Allen Probes EFW instrument. However, the methodology is certainly repeatable for other spacecraft missions that measure both the electric and magnetic wave fields. This technique therefore permits for a reduction in the uncertainties associated with electric field measurements due to the variable instrument-plasma interface.

2. Instrument-Plasma Coupling Effects

The formation of a plasma sheath around spherical double probe electric field sensors can attenuate the output voltage, causing under-measurements of the electric field wave power spectral density. A simple way to model this is to represent the coupling of the antenna to the plasma by a voltage divider with complex impedance. The antenna is considered connected to the plasma through the parallel combination of a capacitor and a resistor in the sheath region as shown in Figure 1 [6, 7].

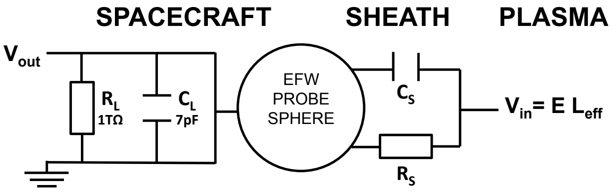

Figure 1. Voltage divider circuit used to represent an EFW electric field antenna immersed in a plasma. R_S and C_S are the sheath resistance and capacitance, respectively. R_L and C_L are the load resistance and capacitance, respectively.

For the Van Allen Probes EFW, the load capacitance is estimated as 7 pF and the load resistance is estimated as 1 TOhm. This configuration yields the sheath impedance function (ratio of output voltage to input voltage) shown in Equation 1.

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_{out}}{E\ L_{eff}}\right| = \left[1 + \frac{R_S}{R_L}\left(\frac{1 + j\omega R_L C_L}{1 + j\omega R_S C_S}\right)\right]^{-1} \quad (1).$$

Where V_{in} and V_{out} are the input an output voltages respectively, E is the electric field, L_{eff} is the effective length (typically equal to the probe separation for spherical double probes), j is the imaginary unit, ω is the angular frequency (2πf), R_S and C_S are the sheath resistance and capacitance respectively, and R_L and C_L are the load resistance and capacitance respectively.

3. Cold Plasma Dispersion Relation

To quantify the signal attenuation that arises due to the formation of a plasma sheath around the electric field sensors, the electric field measured by the Van Allen Probes is compared to the electric field wave power predicted by cold plasma theory (using observations of the magnetic field, B) for whistler-mode waves. The predicted electric field, E, is calculated using Equation 2.

$$E^2 = \frac{c^2}{n^2}\left(\frac{a(b+1) + (n^2\cos\theta\sin\theta)^2}{a\ b + P^2\cos^2\theta}\right)B^2 \quad (2).$$

Where *c* is the speed of light, *n* is the refractive index, *θ* is the wave normal angle, and;

$$a = (P - n^2\sin^2\theta)^2 \quad (3).$$

$$b = \left(\frac{D}{S - n^2}\right)^2 \quad (4).$$

$$P = 1 - \frac{f_{pe}^2}{f} \quad (5)$$

Considering the ratio between this calculated electric field and the electric field measured by the Van Allen Probes, as a function of both frequency and density, reveals a structure consistent with signal attenuation due to the variable instrument-plasma interface. Figure 2 shows the median of the logarithm of the ratio between the observed and calculated electric field wave power spectral density.

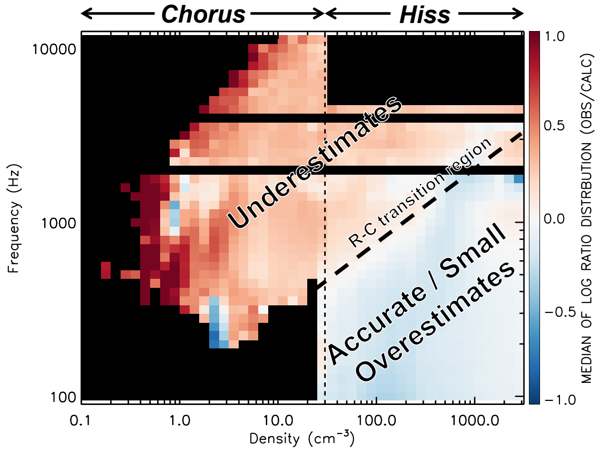

Figure 2. Median logarithm of the ratio between observed and calculated electric field power spectral densities.

4. Building the Sheath Impedance Model

This instrument-plasma coupling effect can subsequently be quantified by fitting functions of the form of Equation 1 to the square root of the ratios shown in Figure 2 (ratio of wave amplitudes), for each density bin. Since the values associated with the spacecraft side of the circuit are well established, the free parameters for the fits are R_S and C_S (we initially select to fix L_{eff} = 1, meaning that the effective length is equal to the probe separation). For each density interval, sheath resistance values are varied between 1 and 1000 MOhm and sheath capacitance values are varied between 1 and 20 pF (expected range of values for the on-orbit plasma conditions), until the values that minimize the chi-squared statistic are identified. If the sheath resistance and capacitance values that minimize chi-squared are not at the extremes of the permitted values, and the fit yields a reduced chi-squared value less than 0.5, the fits are deemed to reflect the variations observed in the data.

The sheath resistance and capacitance values that yield good fits to the data are then studied as a function of plasma density. It is found that for the majority of the fits, there can be a quite a large degree of variability in both the sheath resistance and sheath capacitance values before the fits are deemed to no longer represent the data. Thus, in order to approximate the effect of instrument-plasma coupling, simple fits are performed to the sheath resistance and capacitance values as a function of plasma density, N. These fits yield Equations 6 and 7.

$$\log_{10} R_S = 1.948 - 0.4925 \times \log_{10} N \qquad (6).$$

$$C_S = 8.730 + 1.164 \times \log_{10} N \qquad (7)$$

Where N is in cm^{-3}, R_S is in MOhms and C_S is in pF.

Using the parameters calculated from Equations 6 and 7, and inserting them in to Equation 1, permits the calculation of a density and frequency dependent correction factor for the instrument-plasma coupling effects associated with the Van Allen Probes electric field spherical double probe sensors.

These correction factors have been thoroughly tested in order to assess their accuracy. While these correction factors take initial strides towards determining the variable probe-plasma coupling impedance, they are not able to account for all of the variability. In particular, at densities between ~30 cm^{-3} and 2000 cm^{-3} and at frequencies between 100 Hz and 1 kHz, where the observed electric field is generally greater than expected, an effect that can not be accounted for with this initial sheath impedance correction. Investigation of these anomalous gains reveals that they may be attributed to the shorter spin axis antennas, and potentially associated with the lower hybrid resonance. It is also noted that in a very low-density plasma, where the Debye length is comparable to the separation of the spherical double probes, the 'shorting effect' may occur [8]. This means that the effective length may no longer be considered to be simply the separation between the spherical double probes of the EFW ($L_{eff} = 1$), and is actually some fraction of this length ($L_{eff} < 1$).

In order for these additional effects to be accounted for in the sheath impedance model, we once again conduct minimized chi-squared fits of the sheath impedance function to the median wave amplitude ratios. This time, the values for the sheath capacitance and sheath resistance are obtained by Equations 6 and 7, with a new parameter introduced to describe the relative effective length (effective length divided by the probe separation, L_{eff}). This value is permitted to vary between 0.5 and 1.5. Values less than unity allow for the shorting effect to be accounted for at low densities, whereas values greater than unity allows for the model to account for the gains from the spin axis antenna.

The values of the relative effective length that minimize chi-squared are then considered as a function of density. Unlike the sheath capacitance and resistance values, the relative effective length values do not follow a simple functional form. As such, a smoothed function is manually generated to describe its variability. Table 1 provides the smoothed relative effective length values that may be interpolated between in order to generate this function, allowing for the relative effective length to be obtained at any density. The sheath capacitance, sheath resistance, and relative effective length values may now be used to provide an improved estimate of the sheath impedance function of the EFW spherical double probes instrument. Note that although the source of the anomalous gain has been identified as the spin axis antennas, fitting of sheath impedance functions is performed to the total wave amplitude ratios (sum over all three antennas), and as such, the sheath impedance correction factor is applied to the total wave power amplitude also.

Table 1. Relative effective length values that minimize the chi-squared statistic as a function of plasma density.

N (cm^{-3})	*L_{eff}*
0.1	0.5
0.5	0.5
1.0	0.6
2.0	0.9
5.0	1.0
30	1.0
100	1.2
1,000	1.2
2,000	1.0
10,000	1.0

5. Testing the Sheath Impedance Model

With a sheath impedance model for the Van Allen Probes EFW instrument now fully developed, we can now test its accuracy in accounting for signal attenuation in the measured total electric field wave power. This is done by comparing the total sheath-corrected wave electric field to the total wave electric field calculated from the magnetic field and the full cold plasma dispersion relation (Equation 2). It is assumed that the calculated electric field is the real electric field prior to any signal attenuation. The ratio between the two (real electric field wave power divided by the sheath-corrected observed electric field wave power) provides a metric for determining the accuracy of the sheath correction factor.

Figure 3 (top) shows the median uncorrected total wave power ratio as a function of density and frequency. Red bins indicate that the average observed electric field wave power is less than the average real electric field by a factor ≥ 1.5, green bins indicate that the average observed wave power is within a factor of 1.5 of the average real electric field, and blue bins indicate that the average observed electric field wave power is greater than the average real electric field by a factor ≥ 1.5. The percentages of bins of each color are also listed. Figure 3 (bottom) shows the same parameters but with the sheath correction now applied. It is evident that applying the sheath impedance correction results in a significant improvement in agreement of observed and real wave powers. Prior to any sheath correction, only 39% of bins contained an average total electric field wave power that was in agreement (within a factor of 1.5) with the real electric field. Using the sheath correction determined in this study increases this value from 39% up to 88%.

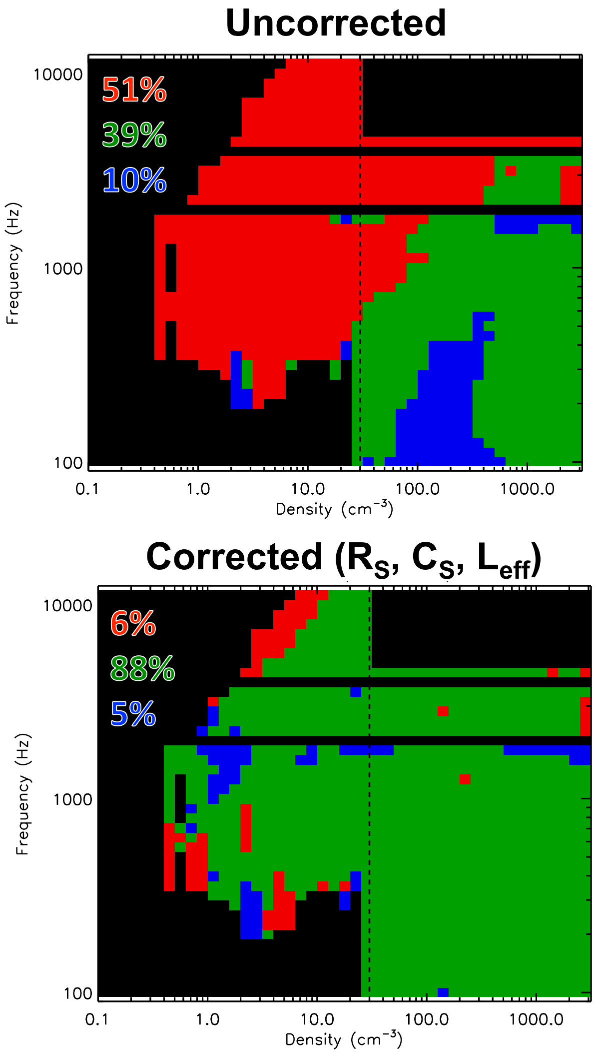

Figure 3. Median wave power ratios parametrized as; the average observed electric field wave power is less than the average real electric field by a factor of 1.5 or greater (red), the average observed wave power is within a factor of 1.5 of the average real electric field (green), or the average observed electric field wave power is greater than the average real electric field by a factor of 1.5 or greater (blue). Top) Uncorrected, Bottom) Corrected using R_S, C_S, and L_{eff} values from the sheath impedance correction.

6. Conclusions

Using comparisons between the electric field wave power spectra predicted by cold plasma theory (using magnetic field observations), and the electric field wave power measured by the EFW spherical double probes instrument, a simple density-dependent model has been developed in order to quantify the effect of the variable instrument-plasma interface. This model also accounts for the antenna-shorting factor, as well as the anomalous gains observed in the spin axis antennas under certain conditions. This model has been demonstrated to be successful in significantly improving agreement between calculated and observed power spectra and wave powers. The sheath impedance model presented here allows for all electric field wave observations made by the Van Allen Probes EFW instrument to be corrected for the variable antenna-sheath impedance. The methodology used in this study may also be applied to other spacecraft missions to quantify, and correct for, instrument-plasma coupling effects on other electric field instruments. This paper summarizes the methodology and results of previously published materials [9, 10].

7. References

1. J. R. Wygant et al., “The Electric Field, and Waves (EFW) instruments on the Radiation Belt Storm Probes Mission”, Space Sci. Rev., 2013, doi:10.1007/s1124-013-0013-7.

2. L. R. O. Storey, “The design of an electric dipole antenna for VLF reception within the ionosphere”, Tech. Rep. 308TC, July 1963.

3. U. Fahleson, “Theory of electric field measurements conducted in the magnetosphere with electric probes”, Space Sci. Rev., 7, 238, 1967

4. H. C. Koons, D. A. McPherson, and W. B. Harbridge, “Dependence of very-low-frequency electric field antenna impedance on magnetospheric plasma density”, J. Geophys. Res., 1970, doi:10.1029/JA075i013p02490.

5. C. A. Kletzing et al., “The Electric and Magnetic Field Instrument Suite and Integrated Science (EMFISIS) on RBSP”, Space Science Reviews, 2013, doi:10.1007/s11214-013-9993-6.

6. M. H. Boehm et al., “Wave rectification in plasma sheaths surrounding electric field antennas”, J. Geophys. Res., 99(A11), 1994, doi:10.1029/94JA01766.

7. D. A. Gurnett, “Principles of Space Plasma Wave Instrument Design, in Measurement Techniques in Space Plasmas Fields”, AGU, Washington, D.C. 1998, doi: 10.1002/9781118664391.ch14

8. A. F. Pedersen et al., “Electric field measurements in a tenuous plasma with spherical double probes, in Measurement Techniques in Space Plasmas: Fields,” Geophysical Monograph 103, AGU, 1998.

9. D. P. Hartley et al., “Using the cold plasma dispersion relation and whistler mode waves to quantify the antenna sheath impedance of the Van Allen Probes EFW instrument”, J. Geophys. Res. Space Physics, 121, 2016 doi:10.1002/2016JA022501.

10. D. P. Hartley et al., “An improved sheath impedance model for the Van Allen probes EFW instrument: Effects of the spin axis antenna”, J. Geophys. Res. Space Physics, 122, 2017, doi:10.1002/2016JA023597.