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Revisiting the Singularity Expansion Method for modern electromagnetic problems

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Extended Abstract

In this talk I show how the Singularity Expansion Method, originally developed for radar scattering applications, is an excellent approach to solving problems in the fields of metamaterials and nanophotonics. The Singularity Expansion Method (SEM) was developed from the 1970's [1], enabling the time-domain features of scattering from an antenna or radar target to be represented as a sum of damped sinusoidal terms. It utilizes an integral operator to solve the scattering problem, and a search is performed to find the poles of this operator in the complex frequency plane. This can be regarded as a form of modal analysis for open resonators, where energy is not strongly confined, and radiation losses cannot be treated as a small perturbation. In recent times the SEM has fallen out of favour, with the theory of characteristic modes [2] being the preferred technique for modal analysis of open systems within the field of antenna engineering. These characteristic modes are appealing, because they form an orthogonal set, and existing computational electromagnetic software packages can be readily modified in order to calculate them. However, their connection to the physics of the problem is less clear, since the set of characteristic vectors is different for each frequency.

In an entirely independent development, the physics community has conducted significant research on the theory of quasi-normal modes [3] (also known as the resonant state expansion). This approach is ideal for solving problems such as the emission rate of a molecule or quantum dot located near a scattering object or within an open cavity. Conventional approaches based on normal modes fail in such cases, since the normalisation integrals over the modal field become unbounded. The quasi-normal modes are a non-orthogonal set of modes, satisfying the radiation boundary condition. They are typically found using the finite element method, which leads to some ambiguity associated with the perfectly matched layer (PML) boundaries. They have been successfully used to solve cavity perturbation problems, to demonstrate strong coupling and superradiance and to show the spectral features of complex nanophotonic objects. While these problems can be solved by direct numerical computation, developing a modal picture enables the physics to be much better understood.

Recently, I have shown how the singularities of the integral operator found in the SEM correspond directly to the quasi-normal modes of a system [4]. Moreover, this integral operator approach is highly advantageous, since radiation boundary conditions are naturally taken into account, and spurious modes introduced by the PML boundaries are avoided. A robust procedure is utilized to find the location of these modes within the complex plane. The non-orthogonality of the modes, although counter-intuitive, is an essential feature of an open system. I consider a nano-scaled dielectric disk, as typically used in dielectric metamaterials for the near-infrared spectral range. It is shown that the non-orthogonality corresponds to an energy exchange between modes, giving rise to interesting spectral features such as Fano resonances, or anapolar states, with supressed dipolar radiation. This enables intuitive yet rigorous physical models to be constructed for almost arbitrary electromagnetic structures.

References

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