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New Propagation Regimes of TE Waves in a Waveguide filled with a Nonlinear Dielectric Metamaterial

Yury Shestopalov(1), Eugene Smolkin*(2), and Maxim Snegur(2)
(1) Academy of Technology and Environment, University of Gävle, Gävle, Sweden
(2) Department of Mathematics and Supercomputing, Penza State University, Penza, Russia

Abstract

We consider propagation of surface TE waves in a circular metal-dielectric waveguide filled with nonlinear (Kerr nonlinearity) metamaterial medium. Analysis is reduced to solving a nonlinear transmission eigenvalue problem for an ordinary differential equation; eigenvalues of the problem correspond to propagation constants of the waveguide. For the numerical solution, a method is proposed based on solving an auxiliary Cauchy problem (a version of the shooting method). As a result of comprehensive numerical modeling, new propagation regimes are discovered.

1 Introduction

Analysis of wave propagation in an open metal–dielectric waveguides constitutes an important class of vector electromagnetic problems. A conducting cylinder covered by a concentric dielectric layer, the Goubau line (GL) shown in Fig. 1 is the simplest type of such guiding structures. A complete mathematical investigation of the spectrum of symmetric surface modes in a GL is performed in [1]. Papers [2, 3] develop the theory of wave propagation in layered nonlinear dielectric waveguides. However, there are virtually no results concerning the analysis of wave propagation in a GL with an external concentric layer having nonlinear metamaterial permittivity. We aim this study to fill this gap and consider the problem of electromagnetic TE (transverse-electric) wave propagation in a GL with nonlinear metamaterial permittivity of the dielectric medium filling the GL layer. We consider only the intensity-dependent permittivity. The determination of TE waves is reduced to a nonlinear transmission eigenvalue problem for Maxwell's equations, which is then reduced to a system of nonlinear ordinary differential equations.

2 Statement of the problem

Let us consider three dimensional space $\mathbb{R}^3$ with cylindrical coordinate system $O\rho\varphi z$. The space is filled with an isotropic medium of dielectric permittivity $\varepsilon_c^2\varepsilon_0 = const$, where ε_0 is the permittivity of free space, without sources. The medium is assumed to be isotropic and nonmagnetic. A GL with a cross-section

$$\Sigma := \{(\rho,\varphi,z) : a \leqslant \rho \leqslant b, 0 \leqslant \varphi < 2\pi\}$$

with a generating line parallel to the axis Oz is placed in $\mathbb{R}^3$. It is supposed that everywhere $\mu = \mu_0$, where μ_0 is the permeability of free space.

The geometry of the problem is shown in Fig. 1. The waveguide is unlimitedly continued in z direction.

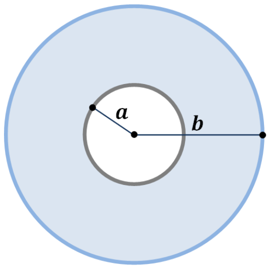

Figure 1. Geometry of the problem. The cross section of the waveguide, which is perpendicular to its axis, consists of two concentric circles of radii a and b: a is the radii of the internal (perfectly conducting) cylinder, and $b-a$ is the thickness of external (dielectric) cylindrical shell.

Let $\mathbf{E}$ and $\mathbf{H}$ be complex amplitudes of an electromagnetic field. The complex amplitudes $\mathbf{E}$, $\mathbf{H}$ satisfy Maxwell's equations

$$\begin{cases} \operatorname{rot}\mathbf{H} = -i\omega\varepsilon\mathbf{E}, \\ \operatorname{rot}\mathbf{E} = i\omega\mu\mathbf{H}, \end{cases} \quad (1)$$

have continuous tangential field components on the media interface $\rho = a$, $\rho = b$ and obey the radiation condition at infinity; i.e., the electromagnetic field decays exponentially as $\rho \to \infty$ in the region $\rho > b$; ω is the circular frequency.

Let us consider TE-polarized waves in the harmonic mode (see [4]),

$$\mathbf{E}e^{-i\omega t} = e^{-i\omega t}(0, E_\varphi, 0)^T, \ \mathbf{H}e^{-i\omega t} = e^{-i\omega t}(H_\rho, 0, H_z)^T,$$

where $\mathbf{E}, \mathbf{H}$ are complex amplitudes and

$$E_\varphi = \mathrm{E}_\varphi(\rho)e^{i\gamma z}, \ H_\rho = \mathrm{H}_\rho(\rho)e^{i\gamma z}, \ H_z = \mathrm{H}_z(\rho)e^{i\gamma z}, \quad (2)$$

where γ is unknown spectral parameter.

Let $k_0^2 := \omega^2\mu\varepsilon_0$. Thus, substituting components (2) into (1) and using the notation $u(\rho;\gamma) := \mathrm{E}_\varphi(\rho)$ we obtain

$$\left(\rho^{-1}(\rho u)'\right)' + \left(k_0^2\tilde{\varepsilon} - \gamma^2\right)u = 0, \quad (3)$$

where $\widetilde{\varepsilon} = \varepsilon_0^{-1}\varepsilon$, and $\widetilde{\varepsilon}$ is defined by formula $\varepsilon = \widetilde{\varepsilon}\varepsilon_0$, where

$$\widetilde{\varepsilon} = \begin{cases} -\varepsilon_m^2 + \widetilde{\alpha} u^2, & a \leqslant \rho \leqslant b, \\ \varepsilon_c^2, & \rho > b, \end{cases} \tag{4}$$

and ε_m^2, ε_c^2 and $\widetilde{\alpha}$ are real constants.

We assume that function u is sufficiently smooth

$$u(\rho) \in C^1\,[a, +\infty) \cap C^2(a,b) \cap C^2(b,+\infty).$$

Let $\kappa^2 := \gamma^2 - k_0^2\varepsilon_c^2$. In the domain $\rho > b$ equation (3) takes the form

$$u'' + \rho^{-1}u' - \rho^{-2}u - \kappa^2 u = 0. \tag{5}$$

In the domain $a \leqslant \rho \leqslant b$ equation (3) takes the form

$$u'' + \rho^{-1}u' - \rho^{-2}u - k^2 u = -\alpha u^3, \tag{6}$$

where $\alpha := k_0^2\widetilde{\alpha}$, $k^2 := k_0^2\varepsilon_m^2 + \gamma^2$.

The necessary solutions to equation (5) must be written in the following form

$$u = CK_1(\kappa\rho), \;\; \rho > b. \tag{7}$$

The function K_1 are the modified Bessel functions and C is constant. The radiation conditions are satisfied because $K_1(k_c\rho) \to 0$ exponentially as $\rho \to \infty$ (see [5]).

Transmission conditions for the functions u and u' result from the continuity conditions for the tangential field components and have the form

$$[u]|_{\rho=b} = 0, \;\; [u']|_{\rho=b} = 0, \tag{8}$$

where $[v]|_{\rho=s} = \lim\limits_{\rho\to s-0} v(\rho) - \lim\limits_{\rho\to s+0} v(\rho)$ is the jump in the limit values of the function at a point s.

We suppose what electric field $\mathbf{E}$ on the boundary $\rho = a$ is defined as follows:

$$u(a) = 0, \tag{9}$$

$$u'(a) = \widetilde{C}. \tag{10}$$

Let us formulate the transmission eigenvalue problem (problem P) to which the problem of surface waves propagating in a GL has been reduced. The goal is to find quantities γ such that, for given $C \neq 0$ (or $\widetilde{C} \neq 0$), there is a nonzero function $u(\rho;\gamma)$ that is defined by formula (7) $\rho > b$ and solves equation (6) for $a < \rho < b$; moreover, the function $u(\rho;\gamma)$ thus defined satisfies transmission conditions (8) for $\rho \in [a, +\infty]$.

The quantities solving problem P are called *eigenvalues*, and the corresponding functions $u(\rho;\gamma)$ are called *eigenfunctions*.

3 Numerical method

The method under consideration makes it possible to find (normalized) propagation constant γ. Consider the Cauchy problem for the system of equations

$$u'' + \rho^{-1}u' - \rho^{-2}u - k^2 u = -\alpha u^3,$$

with the following initial conditions

$$u(a) = 0, \; u'(a) = \widetilde{C}. \tag{11}$$

To justify the solution technique, we use classical results of the theory of ordinary differential equations concerning the existence and uniqueness of the solution to the Cauchy problem and continuous dependence of the solution on parameters.

Using the transmission condition on the boundary b we obtain the following dispersion equation

$$\Delta\left(\gamma,\, u, u'\right) := \kappa u(b) K_1'(\kappa b) - u'(b) K_1(\kappa b), \tag{12}$$

where quantities $u(b)$ and $u'(b)$ are obtained from the solution to the Cauchy problem.

4 Numerical results

We solve numerically the auxiliary Cauchy problem (see [6, 7, 8]) and determine and plot the normalized eigenvalues with respect to the circular frequency.

The following values of parameters are used for calculations: $a = 2$, $b = 4$, $\varepsilon_c^2 = 1$, $\widetilde{C} = 100, \alpha = 0.01$.

Figures 2-4 display the plots of $\gamma(\omega)$ for different values of nonlinearity coefficient α and permittivity ε_m are plotted. We call the dependence $\gamma(\omega)$ the dispersion curve (DC).

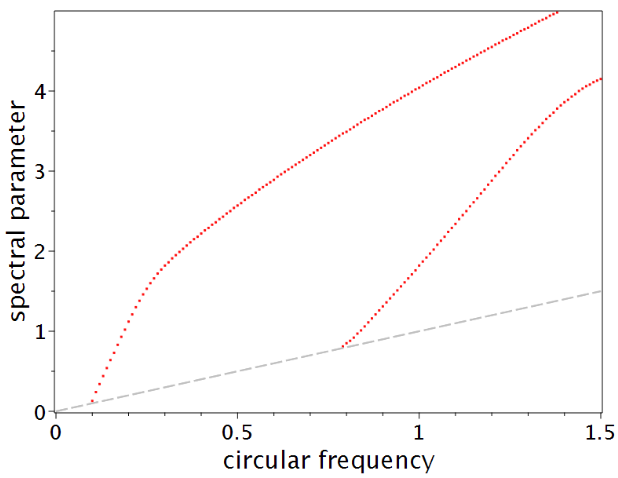

Figure 2. DCs for nonlinear TE waves. The following values of parameters are used for calculations: $\varepsilon_m^2 = -1$.

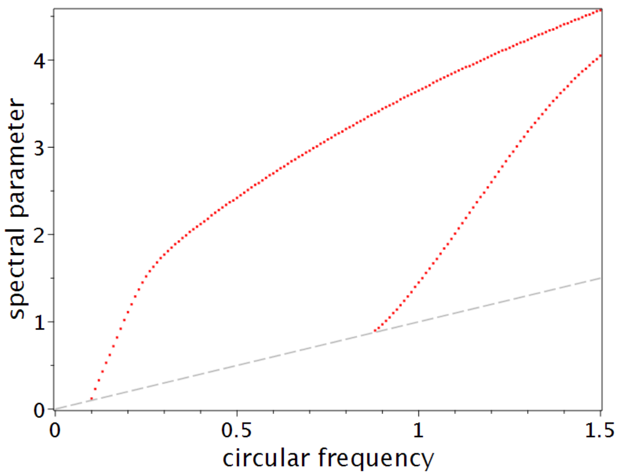

Figure 3. DCs for nonlinear TE waves. The following values of parameters are used for calculations: $\varepsilon_m^2 = -4$.

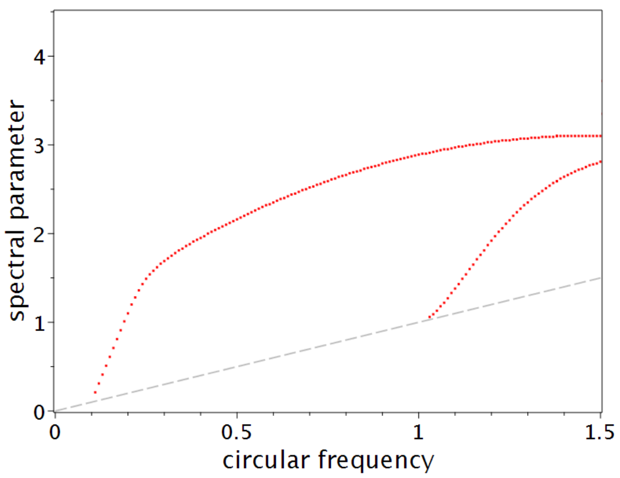

Figure 4. DCs for nonlinear TE waves. The following values of parameters are used for calculations: $\varepsilon_m^2 = -9$.

5 Conclusion

The use of the proposed version of the shooting method may be justified for the analysis of nonlinear metamaterial waveguide. For such problems an explicit dispersion equation is not available and numerical investigation of the eigenvalue spectrum can be carried out only by a specific method developed in this paper.

And finally, we determined numerically nonlinear TE waves propagating in a nonlinear metamaterial waveguide and discovered 'new' eigenvalues that are not perturbations of eigenvalues of the linear problem. These eigenvalues correspond to a new (purely nonlinear) propagation regime. Whether these mathematically predicted purely nonlinear waves really exist, is a hypothesis that can be proved or disproved in physical experiment.

The proposed numerical method enables finding all eigenvalues and proves to be efficient in the case of discrete eigenvalues.

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