



Fast Satellite Selection Techniques and DOPs for Multi-GNSS Positioning

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Abstract

GNSS receivers are designed to track more of satellites corresponding to multi-constellation (GPS, GLONASS and Galileo). As the position error is function of DOP, which is a multiplicative factor, selection of subset of satellites with minimal Dilution of Precision (DOP) is crucial. Further, due to possibility of tracking more than 20 satellites, fast satellite selection techniques that provide optimal Dilution Precision DOP with less computational load are essential. In this paper, three prominent fast satellite selection techniques namely Quasi Optimal, Recursive Quasi-optimal and are evaluated for multi-constellation.

$$A = \begin{bmatrix} \frac{x_u - x_{s_1}}{\rho_1} & \frac{y_u - y_{s_1}}{\rho_1} & \frac{z_u - z_{s_1}}{\rho_1} & 1 \\ \frac{x_u - x_{s_2}}{\rho_2} & \frac{y_u - y_{s_2}}{\rho_2} & \frac{z_u - z_{s_2}}{\rho_2} & 1 \\ \dots & \dots & \dots & \dots \\ \frac{x_u - x_{s_n}}{\rho_n} & \frac{y_u - y_{s_n}}{\rho_n} & \frac{z_u - z_{s_n}}{\rho_n} & 1 \end{bmatrix} = \begin{bmatrix} LOS_1 & 1 \\ LOS_2 & 1 \\ \dots & \dots \\ LOS_n & 1 \end{bmatrix}$$

1. Introduction

The performance of GNSS is not only affected by systematic error but also because of user satellite geometry i.e. Geometric Dilution of Precision (DOP). Traditional techniques used for selection of subset of satellites include highest elevation satellite selection algorithm, Kihara’s maximum volume method and Four-step satellite selection technique etc. The above said techniques had limitations in terms of Floating point operations (FLOPs) and in some techniques though availability of satellites are in good number, still only a minimum of four satellites can be considered for DOP estimation. Therefore, fast satellite selection (FSS) techniques are being developed. However, it is also important to obtain optimal DOP values.

2. DOP computation and rating

The Geometric DOP is square root of variances of receiver position estimate in east, north, up component and receiver clock offset [1].

$$\sigma_G = \sqrt{\sigma_E + \sigma_N + \sigma_U + \sigma_{dt}} \tag{1}$$

$$= \sigma \text{tr}\{ (A^T A)^{-1} \}$$

DOP is computed from the design matrix ‘A’ elements (Eq.), which contains X, Y, Z (ECEF) coordinates of satellite vehicles visible at an instant of time. The co-factor matrix (Q_X) is expressed as,

$$Q_X = (A^T A)^{-1} \tag{2}$$

The design matrix ‘A’ is given as,

where,

x_u, y_u, z_u and $x_{s_{1..n}}, y_{s_{1..n}}, z_{s_{1..n}}$ are receiver and satellite positions in ECEF coordinates.

$\rho_{1..n}$ are the respective pseudoranges from satellite to the receiver/user.

The subscript ‘X’ in Eq.(2) signifies the result in ECEF coordinate system. Q_X is a [4x4] matrix and the elements of the matrix are as follows,

$$Q_X = \begin{bmatrix} q_{xx} & q_{xy} & q_{xz} & q_{xt} \\ q_{xy} & q_{yy} & q_{yz} & q_{yt} \\ q_{xz} & q_{yz} & q_{zz} & q_{zt} \\ q_{xt} & q_{yt} & q_{zt} & q_{tt} \end{bmatrix} \tag{3}$$

The diagonal elements of the Q_X matrix are used to compute following DOPs:

Geometry DOP (GDOP) : $\sqrt{q_{xx} + q_{yy} + q_{zz} + q_{tt}}$ (4)

Position DOP (PDOP) : $\sqrt{q_{xx} + q_{yy} + q_{zz}}$ (5)

Time DOP (TDOP) : $\sqrt{q_{tt}}$ (6)

These DOPs (GDOP, PDOP and TDOP) are expressed in equatorial plane. In order to define HDOP and VDOP, the transformation of ‘Q_X’ matrix to local coordinates (n, e, u) is essential. “The factor that multiplies error in ranges to give approximate error in position is PDOP” (Strang and Borre, 1997). The dilution of precision in horizontal (two dimension), vertical (one dimension) and with respect to time are denoted as HDOP, VDOP and TDOP respectively. Table 1 depicts the rating for geometric DOP values [2].

Table 1 Geometric DOP Rating

DOP	Rating
1	IDEAL
2-4	EXCELLENT
4-6	GOOD
6-8	MODERATE
8-20	FAIR
20-50	POOR

3. Significance of geometric DOP

Fig.1 depicts relationship between satellite geometry and accuracy of user position.

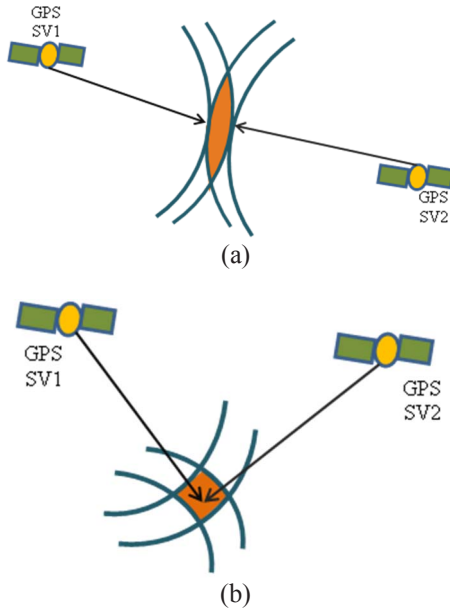


Figure 1 Relationship between satellite geometry and accuracy of user position

The uncertainty bounds of user position is expected to lie in the area of intersection by two circles. Fig.2a and b shows the uncertainty bounds of position solution. When LOS vectors are perpendicular, position error will be reduced. Theoretically, when low-elevation satellites are distributed evenly in azimuth, it lead to better horizontal accuracy, higher elevation satellites contribute to better vertical accuracy.

4. Fast Satellite Selection Techniques

This section describes the three fast satellite selection techniques used for the present study.

4.1 Quasi-optimal technique

Quasi-optimal techniques is proposed by Park and How (2001). The technique is capable of considered more than 4 satellites for DOP computation. The cost for the i^{th} measurement of a particular satellite is the sum of the costs between the i^{th} measurement and all other 'n' measurements ($j=1,2,\dots,n$) and is expressed as [3],

$$CF_i = \sum_{j=1}^n \cos 2 \theta_{ij} = \sum_{j=1}^n (2 \cos^2(\theta_{ij}) - 1) \quad (8)$$

In this way, the row and column corresponding to the maximum cost are eliminated.

The elimination of highest cost corresponds to a particular satellites. Depending upon the availability of satellites, number of satellites that be ignored from the subset can be decided.

4.2 Recursive quasi-optimal technique

With 'n' visible satellites at an epoch in recursive quasi-optimal technique for, the GDOP is calculated for nC_{n-r} ($r=1,2$ or more) combinations/subsets. The following steps are performed by the technique to reduce computational load [4],

1. For the all visible satellites 'n', co-factor matrix is defined, $Q_n = A^T A$.
2. Subsets are formed using nC_{n-1}
3. Each subset is checked for omitted/ignored satellite, of which LOS vector is given as,
$$L_i = [(x_i, y_i, z_i), 1]$$
4. Subtract from cofactor matrix ' Q_n ' the result of ' $L_i^T L_i$ ' and take the trace of resultant matrix ' $Q_{k,i}$ ' for GDOP.
5. For all the subsets (step 2) generated, Step 3 and 4 are implemented. Subset with minimum GDOP is considered.
6. Step 2 - 5 are repeated until the desired number of satellites in the subset is obtained and then GDOP is calculated.
7. The total number of iterations at an epoch in this technique are $n - k_{sb}$, where, 'n' is the total number of visible satellites at an epoch and ' k_{sb} ' is the desired number of satellites in a subset.

4.3 Fast Satellite Selection algorithm

A new fast satellite selection technique based on Newton's identities for GDOP calculation is proposed by Fanchen et. al. (2013). In this algorithm, first the elevation angles are divided into 3 ranges; low ($5^\circ-30^\circ$), medium ($30^\circ-65^\circ$) and high ($65^\circ - 90^\circ$). Then the visible satellites are sorted in descending order based on their elevation angles. On iteration basis a maximum of 6 satellites are chosen. The satellite with the largest elevation angle is chosen first and so on. when more than 3 satellites are available in low region, then the next satellite with the largest azimuth difference (approximate greater than 60°) is chosen. The fourth satellite is selected such that it contributes to

smallest GDOP, which is from elevation range (30°-65°), smallest one selected first. The fifth and sixth satellites are selected based on the smallest GDOP (based on Newton's Identities) [5].

5. Experimental setup and Data acquisition

Multi-frequency GNSS receiver of make Septentrio, Nv (Model: PolaRxs pro) capable of tracking GNSS (GPS, GIONASS, Galileo) and SBAS (WAAS, GAGAN, EGNOS) satellite signals was setup at Geethanjali College of Engineering and Technology (GCET), Hyderabad, India. Fig.3 depicts the antenna at top of the building and the receiver connected to HP workstation for continuous data acquisition. The raw data is recorded in Rinex format (Version 3.0). Two files both the observation file and navigation file are obtained for analysis. The data sampling interval is 30s is used. Data corresponding to March 2017 is considered for the analysis. The satellite constellations considered include GPS, GLONASS and Galileo.

6. Results and Discussion

The visibility of total number of satellites for a typical day at GCET site for three GNSS (GPS, GIONASS and GALILEO) is minimum 15 and a maximum of 24. Since the geometric DOP is combination of various DOPs (HDOP, VDOP, PDOP and TDOP), their values obtained from the three fast satellite selection techniques considered, are compared to understand the performance of the methods.

Fig.2, 3 and 4 depicts the DOPs due to 'Recursive Quasi-Optimal', 'Quasi-Optimal' and 'FSS based on newtons identity'.

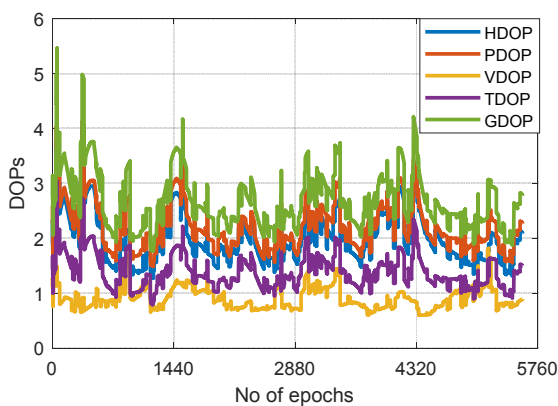


Figure 2 Quasi-optimal DOP estimation

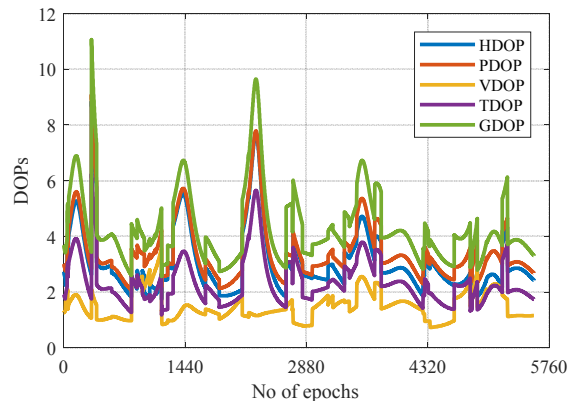


Figure 3 FSS Newton's identity DOP estimation

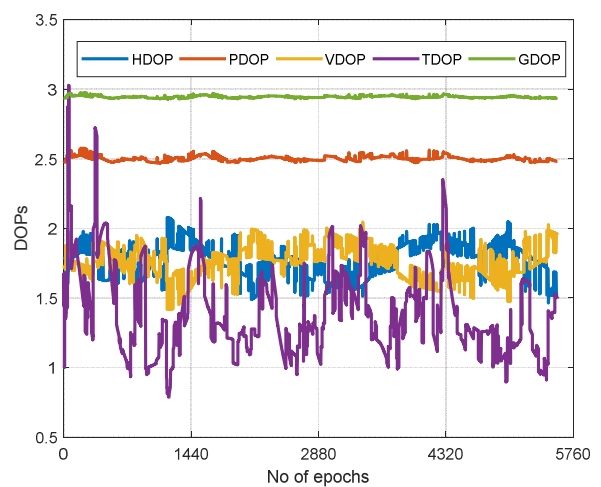


Figure 4 Recursive-Quasi Optimal DOP estimation

Table 2, 3 and 4 shows that lowest mean HDOP is 1.77 for Recursive-Quasi optimal technique compared to other two methods. Also the maximum VDOP and HDOP is 2. From Table 3 it can be noticed that maximum values of DOPs; HDOP, VSOP, PDOP, TDOP and GDOP are 8.83, 3.56, 9.07, 6.22 and 11.06 respectively for FSS based newtons identity technique. This technique is not giving an optimal choice of DOP values. This could be due to constrain on selection of maximum 6 satellites for DOP computation. Though computational faster compared to other two techniques, performance is poor, as observed from Table (3) in multi-constellation case. Among all the three techniques Recursive-Quasi optimal method is giving optimal DOP values. The maximum GDOP is less than 3.

Table 2 Descriptive statistics of Quasi-optimal

Quasi-Optimal technique	Minimum	Maximum	Mean
HDOP	1.27	4.25	1.99
VDOP	0.60	1.87	0.91
PDOP	1.48	4.50	2.21
TDOP	0.78	3.02	1.38
GDOP	1.76	5.46	2.67

Table 3 Descriptive statistics of FSS-Newton's identity

FSS-Newton's identity	Minimum	Maximum	Mean
HDOP	1.7	8.83	3.05
VDOP	0.72	3.56	1.49
PDOP	2.03	9.07	3.46
TDOP	1.17	6.22	2.36
GDOP	2.55	11.06	4.32

Table 4 Descriptive statistics of Recursive-Quasi Optimal

Recursive-Quasi Optimal	Minimum	Maximum	Mean
HDOP	1.46	2.07	1.77
VDOP	1.41	2.04	1.75
PDOP	2.64	2.57	2.50
TDOP	0.78	3.02	1.38
GDOP	2.92	2.97	2.94

7. Conclusions

Three fast satellite selection techniques are evaluated for existing multi-GNSS constellation (GPS, GLONASS and Galileo). The observed values DOP values, shows that 'Recursive Quasi-optimal' technique is giving optimal DOP values. The maximum GDOP due this method is giving a value less than 3. From table 1, as indicated GDOP value between 2-4 is considered as excellent/very good. The salient features of Recursive Quasi technique are low computational load and near-optimal/optimal geometries, thus provide greater accuracy and reliability to multi-GNSS systems.

Acknowledgement

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