URSI AP-RASC 2019, New Delhi, India, 09 - 15 March 2019

Estimation of Ground Footprint for Airborne Antenna Systems

Suman Pradhan*, and Bhaskar Gupta
Dept. of ETCE, Jadavpur University
sumanpradhan.kut2@gmail.com

Abstract

Antennas are mounted on aircrafts for civilian use and also used in missiles and shells for airborne warfare applications. It is important to calculate the coverage area of such antennas for system design. In this work, a formula is derived for getting an estimate of the coverage area i.e. ground footprint of such airborne projectile systems.

1. Introduction

In electronic warfare applications, missiles and artillery shells are used which have a projectile motion. In their downward trajectory they have to detect the target by analyzing the received signal. Sometimes the target has to be identified from the clutter or the clutter is the intended target. This is the principle of proximity fuze [1]. The estimation of this clutter power is very important for system design. The clutter power is dependent on the area illuminated by the antenna system. For this the antenna footprint needs to be calculated.

Footprint calculation for antennas mounted on a satellite has been done in [2] for different antenna patterns. However no such calculation is available for systems having a projectile motion with variable altitudes at low heights where the earth surface can be assumed flat. This work is an effort to mitigate the limitation to help in designing better and more accurate electronic warfare systems.

2. Calculation

Let us consider an antenna or antenna array which is placed such that the radiation is directed towards the axis of the projectile. The antenna system has an omni-directional radiation pattern in the azimuth plane. Let us assume that the system is located at a point O which is at a height H from the ground. It is tilted at an angle α degrees with respect to the ground plane. We also assume that the projectile is oriented along the x-axis. The HPBW of the antenna is assumed to be 2*β degrees in every constant phi plane. The location of the antenna along with the ground plane is shown in fig. 1 below. The area which is illuminated by at least half of the power in the broadside direction is assumed to lie in the antenna footprint.

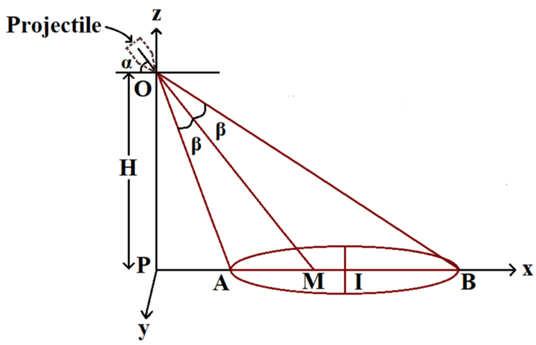

Figure 1. Shape of antenna footprint along with the projectile at O

As the HPBW is same in every constant phi plane, the radiation from the antenna looks like a right circular cone. We know that when a plane intersects the cone at an angle greater than the semi vertical angle of the cone (α>β), the shape formed at the face of the cone is an ellipse [3]. So the problem can be thought as an equivalent problem as shown in fig. 2 below.

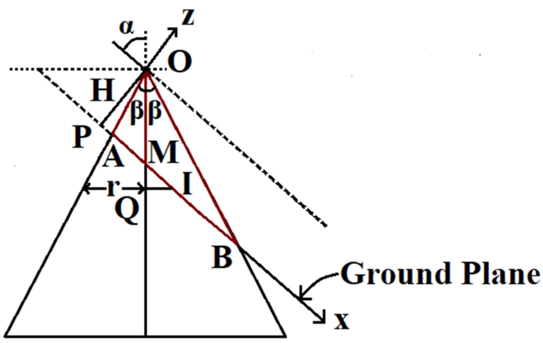

Figure 2. Equivalent representation of the problem

In fig. 2, OP is the perpendicular line segment drawn from the point O to the ground plane. So length of segment OP is the height H. Hence the triangles POA, POM and POB are all right angled triangles. Segment AB is the major axis of the ellipse forming the ground footprint. Its length can be found out by calculating the lengths of segments AP and BP.

From ΔPOA, ∠ POA=90º -α-β.

$$\tan(90^\circ - \alpha - \beta) = AP/OP$$
$$\Rightarrow AP = OP*\tan(90^\circ - \alpha - \beta)$$
$$\Rightarrow AP = H* \tan(90^\circ - \alpha - \beta) \qquad (1)$$

From ΔPOB, ∠ POB=90º-α+β.

$$\tan(90^\circ - \alpha + \beta) = BP/OP$$
$$\Rightarrow BP = OP*\tan(90^\circ - \alpha + \beta)$$
$$\Rightarrow BP = H* \tan(90^\circ - \alpha + \beta) \qquad (2)$$

The major axis of the ellipse is AB which is given by

$$AB = BP - AP$$
$$\Rightarrow AB = H* \tan(90^\circ - \alpha + \beta) - H* \tan(90^\circ - \alpha - \beta)$$
$$\Rightarrow AB = H * \{\tan(90^\circ - \alpha + \beta) - \tan(90^\circ - \alpha - \beta)\}$$
$$\Rightarrow AB = H * \{\cot(\alpha - \beta) - \cot(\alpha + \beta)\} \qquad (3)$$

Point I is the mid-point of segment AB. In an ellipse, the major axis and minor axis bisect each other at their common midpoint. So the minor axis passes through the point I and is perpendicular to the plane of the paper. To find out the length of the minor axis we consider the circle parallel to the base of the cone and passing through the points Q and I with point Q as the centre of the circle. The minor axis lies on the circle and r1 is the semi-minor axis as shown in fig. 3 below.

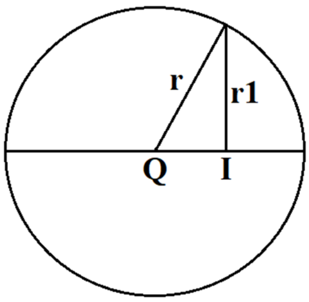

Figure 3. Circle showing the semi-minor axis r1

If we can find out the radius r of the circle and the distance IQ, we can find the length of the semi-minor axis by the following relationship

$$r1 = \sqrt{r^2 - IQ^2} \qquad (4)$$

The radius can be found out if we know the dimension of segment OQ. Segment OQ is the sum of segments OM and MQ.

From ΔPOM, ∠ POM=90º-α.

$$\cos(90^\circ - \alpha) = OP/OM$$
$$\Rightarrow OM = OP/\cos(90^\circ - \alpha)$$
$$\Rightarrow OM = H/ \cos(90^\circ - \alpha)$$
$$\Rightarrow OM = H*\mathrm{cosec}(\alpha) \qquad (5)$$

Both the segments MQ and IQ can be found out if we can find the length of segment MI. Segment MI is

$$MI = AI - AM$$
$$\Rightarrow MI = (AB/2) - (MP - AP) \qquad (6)$$

From ΔPOM, ∠ POM=90º-α

$$\tan(90^\circ - \alpha) = MP/OP$$
$$\Rightarrow MP = OP*\tan(90^\circ - \alpha)$$
$$\Rightarrow MP = H* \tan(90^\circ - \alpha) \qquad (7)$$

Substituting the values of AP, AB and MP from (1), (3) and (7) in (6), we get

$$MI = H * \left\{\frac{\tan(90^\circ - \alpha + \beta) + \tan(90^\circ - \alpha - \beta)}{2} - \tan(90^\circ - \alpha)\right\}$$

$$\Rightarrow MI = H \left\{\frac{\cot(\alpha+\beta)+\cot(\alpha-\beta)}{2} - \cot(\alpha)\right\} \qquad (8)$$

From ΔMIQ, ∠ QMI = α

$$MQ = MI* \cos(\alpha) \qquad (9)$$
$$IQ = MI* \sin(\alpha) \qquad (10)$$

Hence, we can find the radius r according to the relation

$$\tan(\beta) = r / OQ$$
$$\Rightarrow r = OQ * \tan(\beta)$$
$$\Rightarrow r = (OM + MQ) * \tan(\beta)$$
$$\Rightarrow r = \{H * \mathrm{cosec}(\alpha) + MI * \cos(\alpha)\} * \tan(\beta) \qquad (11)$$

Substituting the values of IQ and r from (10) and (11) in (4), we can find the length of the minor axis. The relation thus derived to determine the minor axis is

$$2 * r1 = 2 * H * \sqrt{\left[\mathrm{cosec}(\alpha) + \left\{\frac{\cot(\alpha-\beta)+\cot(\alpha+\beta)}{2} - \cot(\alpha)\right\} * \cos(\alpha)\right]^2 * \{\tan(\beta)\}^2 - \left\{\frac{\cot(\alpha-\beta)+\cot(\alpha+\beta)}{2} - \cot(\alpha)\right\}^2 * \{\sin(\alpha)\}^2} \qquad (12)$$

Thus the area of the ellipse can be found out from the lengths of major axis and minor axis using simple geometry as [4]

$$\text{Area} = \text{Major Axis} * \text{Minor Axis} * pi / 4 \qquad (13)$$

Substituting the values of major axis and minor axis from (3) and (12) in (13), we get

$$\text{Area} = AB*2*r1*pi/4 \qquad (14)$$

3. Conclusion

A general formula for calculating the ground footprint has been obtained with some assumptions like the antenna has an omnidirectional pattern and the tilt of the projectile is greater than half of the half power beamwidth. The formula is likely to be used in warfare applications.

4. Acknowledgements

Suman Pradhan would like to thank CSIR for financial support vide file no. 09/096(0923) 2018 EMR-I. The authors would like to thank ARDE, DRDO for giving us an opportunity to work under a sponsored project.

5. References

1. L. Brown, “The origins of the proximity fuze,” *IEEE MTT-S International Microwave Symposium Digest*, **2**, June 1998, pp. 425-428, doi: 10.1109/MWSYM.1998.705024.

2. K. P. Spies, and E. J. Haakinson, “Calculation of Geo-stationary satellite footprints for certain idealized antennas,” *NTIA Report U.S. Department of Commerce*, October 1980.

3. V. L. Hansen, Shadows of the Circle, *World Scientific Publishing Co. Pvt. Ltd.*, 1998, ISBN: 981-02-3418-X.

4. D. M. Y. Sommerville, Analytical Conics, *G. Bell and Sons*, 3rd Edition, 1961.