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Calculation of the Lightning Induced Response on Overhead Transmission Lines based on the Electric Field Representation Methods

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Abstract

The transient electromagnetic field produced by an indirect lightning can induce to overhead power lines, which may cause serious effects to the power system. When calculating the lightning induced voltages on the overhead lines with the conventional numerical method, to pursue a sufficiently accurate results, the lightning electric field at a large number of observation points along the lines should be calculated. However, since the numerical integration are commonly involved when calculating the lightning electric field, the significant CPU cost is normally required for the whole computational procedure. In order to enhance the computational efficiency, two approaches which based on the electric field representation technique were proposed. In these two methods, the lightning electric fields at several observation points along the line are calculated by using the conventional numerical method first, then the electric field along the entire line can be represented with the continuous functions by using two different fitting methods, which are matrix pencil method (MPM) and the least squares-nonlinear least squares (LS-NLS) method. As a result, the necessary number of observation points where the lightning electric field should be calculated by using the conventional numerical methods is significantly decreased. The proposed methods were validated and compared by some numerical examples.

1. Introduction

Overhead transmission lines are the essential components in the power system. Transient electromagnetic fields produced by an indirect lightning strike may coupling to the overhead transmission lines and inducing the high amplitude of the transient response, which are harmful to the power system [1]. Therefore, in order to protect the power system from indirect lightning, it is important to predict lightning induced response on overhead transmission lines by using a field-to-transmission line coupling model.

Normally, the lightning electromagnetic fields that couple to overhead lines have a non-uniform distribution along the line conductors. In general, to calculate the lightning-induced voltage on overhead line when taking into account ground losses, the common way is by way of a numerical method. In the numerical calculation, the lightning electric fields at several observation points along the overhead line should be evaluated. To obtain a relatively accurate solution, the space discretization along the line should generally be much smaller than the minimum wavelength of the exciting electric field. For lightning electromagnetic pulse, whose frequency spectrum extends to about 10 MHz, a space discretization step of a few meters is required. Since the overhead power lines usually extend to lengths of up to several tens of kilometers, the number of observation points along the line should be very large. However, the general expression for the lightning electric fields involve the integration terms that can not be solved analytically and should be calculated numerically, which are complicated and time consuming. Therefore, to obtain sufficiently accurate results for the lightning-induced response, significant CPU cost is normally required.

To address the above mentioned problem, the electric field representation technique has been proposed [2], in which the horizontal exciting electric fields along the line are represented by the continuous functions, which ensures that the horizontal electric field along the entire line can be evaluated conveniently and efficiently. In this method, the amplitude and phase of the horizontal exciting electric fields are represented by the Fourier series and the Polynomial functions, respectively, using the nonlinear least squares method and the least squares method. However, since the nonlinear least squares method is adopted, the fitting efficiency has the space to be improved. In order to further develop this technique, the matrix pencil method (MPM) [3] is adopted to represent the electric field. In this method, the lightning electric fields along the line are represented by a series of first order exponential functions, which are similar to the case where the exciting electromagnetic field is a uniform plane wave. In this way, the lightning-induced response of the line can be solved analytically with the obtained representation. Moreover, the fitting procedure does not need a starting value or any iterative processes, ensuring that the whole procedure for calculating the lightning-induced response can be done with a high efficiency. This paper would make a comprehensive comparison between these two methods.

The remainder of this paper is organized as follows. Section 2 describes the basic concept of the proposed approaches. Section 3 presents the results associated with several case studies to make the comparison between the different methods. Finally, Section 4 presents the summary and general conclusions.

2. Basic Idea of the Proposed Approaches

2.1 Field-to-Transmission Line Coupling Equations

The field-to-transmission line coupling equations according to the model of Agrawal *et al.* [4] and for a transmission line formed by a single wire above a ground plane are given by:

$$\begin{aligned} &\frac{dV^s(x,j\omega)}{dx} + Z'(j\omega)I(x,j\omega) = E_x^e(x,j\omega) \\ &\frac{dI(x,j\omega)}{dx} + Y'(j\omega)V^s(x,j\omega) = 0 \end{aligned} \qquad (1)$$

where $V^s(x, j\omega)$ and $I(x, j\omega)$ are the scattered voltage and current along the line, respectively, $Z'(j\omega)$ and $Y'(j\omega)$ are the line per-unit-length longitudinal impedance and transverse admittance, respectively, and $E_x^e(x, j\omega)$ is the horizontal component of the exciting lightning electric field along the line conductor.

According to the BLT equations, the solution for the induced voltage at both ends can be expressed by

$$\begin{aligned} V(0,j\omega) &= \frac{-\rho_2(1+\rho_1)S_1(j\omega) - e^{\gamma L}(1+\rho_1)S_2(j\omega)}{\rho_1\rho_2 - e^{2\gamma L}} \\ V(L,j\omega) &= \frac{-e^{\gamma L}(1+\rho_2)S_1(j\omega) - \rho_1(1+\rho_2)S_2(j\omega)}{\rho_1\rho_2 - e^{2\gamma L}} \end{aligned} \qquad (2)$$

where the source terms S_1 and S_2 are

$$\begin{aligned} S_{\mathrm{BLT}}(j\omega) &= \frac{1}{2}\int_0^L e^{\gamma x}E_x^e(x,j\omega)dx - \frac{1}{2}V_1(j\omega) + \frac{1}{2}V_2(j\omega)e^{\gamma L} \\ S_{\mathrm{BLT}}(j\omega) &= -\frac{1}{2}\int_0^L e^{\gamma(L-x)}E_x^e(x,j\omega)dx + \frac{1}{2}V_1(j\omega)e^{\gamma L} - \frac{1}{2}V_2(j\omega) \\ V_1(j\omega) &= -\int_0^h E_z{}^e(0,z)dz, \qquad V_2(j\omega) = -\int_0^h E_z{}^e(L,z)dz, \end{aligned} \qquad (3)$$

where ρ_1 and ρ_2 are the reflection coefficients at the near-end and far-end of the line, respectively, γ is the propagation constant along the line, E_z^e is the vertical component of the exciting lightning electric field, and h is the height of the line.

2.2 The Electromagnetic Field Radiated by a Nearby Lightning

It can be seen that the source terms in the Agrawal *et al.* model include the horizontal component of the lightning electric field along the transmission line and the integral of the vertical component of the lightning electric field at both ends. In the case of a lossy ground, for the horizontal component of the lightning electric field, the Cooray-Rubinstein formula (*e.g.* [5]) has been shown to provide a reasonable approximation.

$$H_{\phi p}(r,z,j\omega) = \frac{1}{4\pi}\int_{-H}^{H} I(z',j\omega)\left[\frac{r}{R^3} + \frac{j\omega r}{cR^2}\right]e^{\frac{-j\omega R}{c}}dz' \qquad (4)$$

$$\begin{aligned} E_{rp}(r,z,j\omega) &= \frac{1}{4\pi\varepsilon_0}\int_{-H}^{H} I(z',j\omega)e^{\frac{-j\omega R}{c}}\left[\frac{3r(z-z')}{j\omega R^5}\right. \\ &\quad \left. + \frac{3r(z-z')}{cR^4} - \frac{j\omega(z-z')}{c^2R^3}\right]dz' \\ R &= \sqrt{r^2 + (z-z')^2} \end{aligned} \qquad (5)$$

$$\begin{aligned} E_x^e(x,y,z,j\omega) &= E_r^e(r,z,j\omega)\cos\theta \\ &= \left[E_{rp}^e(r,z,j\omega) - H_{\phi p}^e(z,0,j\omega)\sqrt{\frac{\mu_0}{\varepsilon_{rg}\varepsilon_0 + \sigma_g / j\omega}}\right]\cos\theta \end{aligned} \qquad (6)$$

2.3 The Electric Field Representation Methods

In many practical cases, the height of the transmission line is relatively modest and the vertical lightning electric field does not significantly vary along the vertical risers. However, the length of the transmission line is generally quite large, and thus, the non-uniformity of the horizontal lightning electric field should not be neglected. As mentioned above in (3), the integration of the horizontal electric field along the line should be calculated in the BLT method. However, in the indirect lightning case, as in (6) shows, the x-component of horizontal electric field could not be integrated analytically and should be integrated numerically. As mentioned above, to obtain an accurate solution, the observation points along the overhead line should be very large, which might be impractical or even impossible.

2.3.1 The Least Squares-Nonlinear Least Squares Method

In order to address the above mentioned problem, the electric field representation techneque was proposed to represent the exciting electric field along the line with a continuous function with a similar form to the plane wave case in the frequency domain. In this method, the x-component of horizontal lightning electric field along the line are expressed as follows

$$E_x^e(x,j\omega) = A'(x,j\omega)e^{i\theta'(x,j\omega)} \qquad (7)$$

where A' is the amplitude of the horizontal electric field, and θ' is the phase of the horizontal electric field.

The amplitude and the phase of the horizontal electric field along the line are fitted by the Fourier series and the Polynomial functions, respectively. Assuming that there are N observation points along the overhead transmission line with a intervals of Δx at locations x=0, Δx, ……, (N-1)Δx. The horizontal lightning electric fields at each observation point in frequency domain can be expressed as

$$\begin{aligned} A'(n\Delta x,j\omega) &\approx \sum_{n_1=0}^{N_1} a_{\omega,n_1}\cos\left(2\pi n_1\frac{n\Delta x}{\chi_{n_1}}\right) + \sum_{n_1=1}^{N_1} b_{\omega,n_1}\sin\left(2\pi n_1\frac{n\Delta x}{\chi_{n_1}}\right) \\ \theta'(n\Delta x,j\omega) &\approx \sum_{n_2=0}^{N_2} p_{\omega,n_2}(n\Delta x)^{n_2}, \quad n = 0,1,2,...,N-1 \end{aligned} \qquad (8)$$

In this method, the phase θ' is fitted using the least squares method, and the amplitude A' is fitted using the nonlinear least squares method, where the fitting procedure is shown in detail in [2]. Since this fitting procedure adopts the least squares method and the nonlinear least squares method, hereafter, it is simply called LS-NLS method. This method ensures that the horizontal electric field along the entire line can be evaluated conveniently and efficiently.

2.3.2 The Matrix Pencil Method

In order to develop the electric field representation technique, a different way that based on the matrix pencil method (MPM) was proposed. In this method, the complex value of the horizontal electric field along the lines are represented by the first order of exponential functions using the MPM directly. Since the fitting procedure only contains many matrix calculation step, the fitting efficiency is improved. Moreover, the fitted function is the first order exponential function which is similar to the uniform plane wave case and can be brought into the analytical iterative method to handle the MTLs case analytically and efficiently. In this method, the horizontal electric fields in frequency domain are estimated as

$$E_x^e(x) \approx \sum_{i=0}^{M-1} R_i e^{S_i x} \tag{9}$$

Assuming that there are N observation points, the horizontal lightning electric fields at each observation point in frequency domain can be expressed as

$$E_x^e(n\Delta x) \approx \sum_{i=0}^{M-1} R_i e^{S_i n\Delta x} = \sum_{i=0}^{M-1} R_i q_i^{\,n} \qquad q_i = e^{S_i \Delta x} \tag{10}$$

The main following tasks is to determine the unknown coefficients of R_i, q_i and M. The fitting process is shown in detail in [3].

By using the MPM, the representation of the horizontal exciting electric field along the overhead line is obtained with the formulation of (10). Then the lightning induced voltage can be calculated analytically.

3. Validation of the Proposed Methods

To validate the proposed method, some examples are presented. In the example, the electric conductivity and the relative dielectric constant of ground are 0.01S/m and 10, respectively. The modified transmission line with exponential decay model is adopted as the lightning channel model. The adopted channel-base current is the sum of two Heidler's functions with the decay constant of 2 km and the return stroke speed of 1.5×10^8m/s. The height of lightning channel is set to 8 km.

The stroke location is alongside the line, and is 50 m away from the left terminal (near-end). The top view of the considered stroke location with respect to the line (1-km long, 10-m high, 1-cm diameter wire) is shown in Fig. 1.

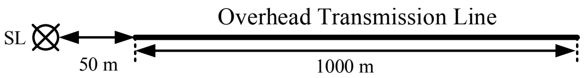

Figure 1. The top view of the considered stroke location.

The x-component of horizontal lightning electric fields are calculated at 11 observation points (N=11) at 100 m intervals by using expression (6). The electric fields at x=0 m, 500 m and 1000 m are shown in Fig. 2. It can be seen that the field exhibits significant non-uniformity along the line length.

Then, the x-component of horizontal lightning electric fields along the line are fitted by the two proposed fitting methods. Since the number of the observation points are limited, to ensure the fitting accuracy, the fitting orders are set to the highest possible value, where the N_1 is set to 4 for the LS-NLS method and the M is set to 5 for the MPM. The fitted results at 900 kHz which obtained from the two methods are shown in Fig. 3. It can be seen that the fitted results obtained from the two fitting methods agree well with the original data. In addition, the fitting errors of the results from the MPM are much smaller than that from the LS-NLS method.

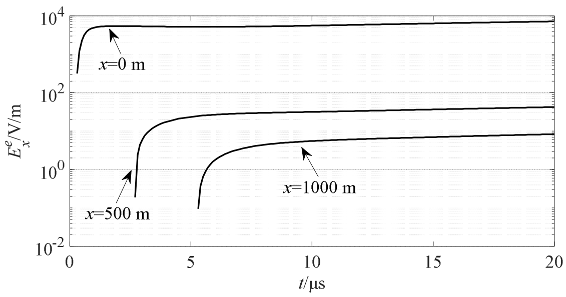

Figure 2. The x-component of the horizontal lightning electric fields at different observation points.

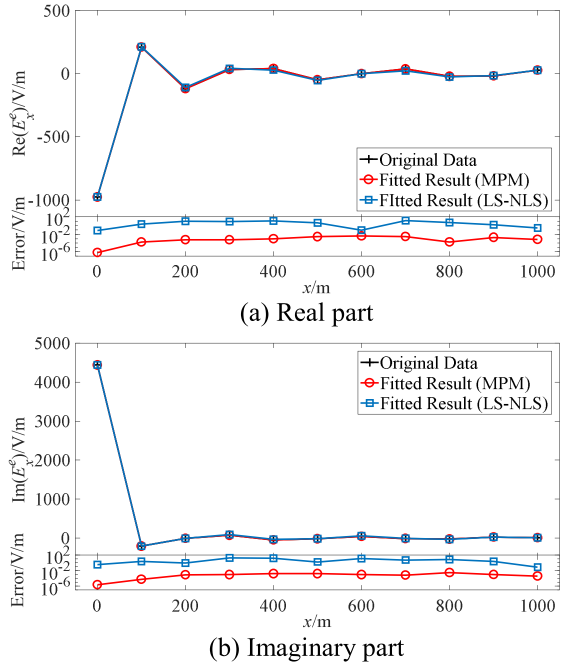

Figure 3. The fitted results at 900 kHz which obtained from the two fitting methods.

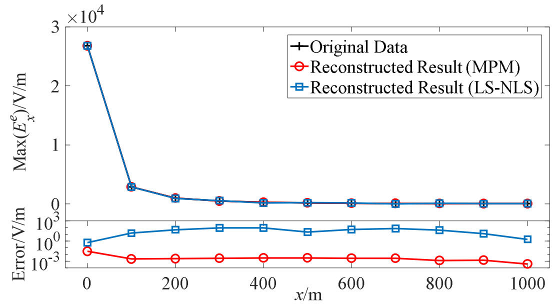

Figure 4. The peak value of the reconstructed time domain waveform of the horizontal electric fields which obtained from the two fitting methods when N=11.

In order to check the fitting accuracy at all the other frequency points, the time domain waveform are reconstructed using the fitted results. The peak value of the reconstructed time domain waveform of the horizontal electric fields at different observation points which obtained from the two fitting methods are shown in Fig. 4. It can be seen that the peak value of the reconstructed horizontal electric fields along the line in time domain agree well with that of the original data. In addition, the error of the reconstructed result obtained from the MPM much smaller than that from the LS-NLS method.

Since the errors in these figures represent the absolute error at different observation points, to make a comprehensive comparison of the fitting error between the two fitting methods, the total error along the entire line should be calculated. To do that, the lightning induced voltage at the near-end is calculated using the two proposed methods. For the MPM, when getting the fitting coefficients, the induced voltage is calculated analytically. And for the LS-NLS method, the x-component of horizontal electric fields at 501 observation points along the line are generated using the obtained fitting coefficients first, then the induced voltage is calculated numerically by using (3). To make a comparison, a reference waveform is calculated by using the conventional numerical method with N=501.

To make the comparison, we set several cases with different number of the observations points N. The proposed two fitting methods and the conventional numerical method are all adopted to calculate the induced voltage at near end. The relationship between the number of the observation points and the relative error of these methods are shown in Fig. 5. It can be seen that with the same observation points, the relative errors of the results obtained from the proposed two methods are all much smaller than that from the conventional numerical method. In addition, the electric field representation technique based on MPM has a higher computational accuracy than that based on LS-NLS method.

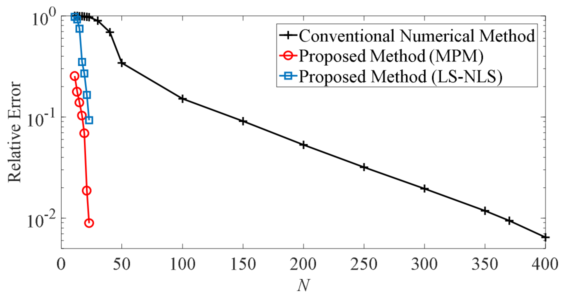

Figure 5. Relationship between the number of the observation points and the relative error of these methods.

The total CPU cost of these methods are also measured. In the calculation, the fitting procedures are applied at 10000 sampling frequency points using the software of Matlab which run on the PC with 2.8 GHz CPU and 16 GB RAM. It is to be noted that the total CPU cost here contains the time spend on numerically calculating the horizontal lightning electric field at N observation points using (6). The relationship between the CPU cost and the number of the observation points N of these methods are shown in Fig. 6. It can be seen that with the same N, the CPU cost of the two proposed method all higher than that of the numerical method, which is due to the additional fitting procedure. In addition, the CPU cost of the MPM method is smaller than that of the LS-NLS method, which means that the MPM has a higher computational efficiency.

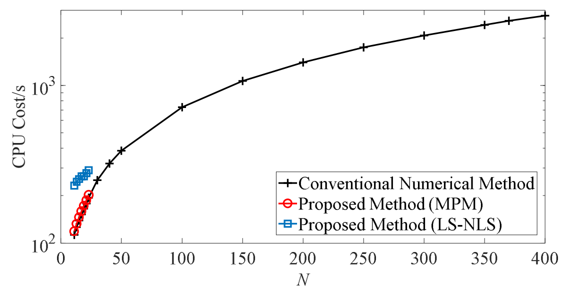

Figure 6. Relationship between the number of the observation points and the CPU Cost of these methods.

4. Conclusions

In this paper, we proposed and compared two methods to handle with the lightning electromagnetic field coupling to overhead transmission lines based on the electric field representation method. In these two methods, the x-component of horizontal lightning electric field at several observation points along the line are calculated using the numerical method first, then the electric field along the entire line can be represented with the continuous functions by using different fitting methods. The proposed methods were validated and compared by several numerical examples. The obtained results showed that with the same number of observation points, the results which obtained from the two proposed method have the much higher accuracy than that from the conventional numerical method. Moreover, with the same number of observation points, the electric field representation method which based on MPM has the both higher computational efficiency and higher accuracy than that based on the LS-NLS method.

5. References

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